Optimum Parameter of Viscous Damper for Damped Adjacent Coupled System

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***Abstract*-In this paper, the dynamic response of two adjacent single-degree-of-freedom (SDOF) structures connected by viscous damper is investigated under base acceleration. The base acceleration is modeled as harmonic excitation as well as stationary white noise random process. The governing equations of motion of the coupled structure are derived and solved for relative displacement and absolute acceleration responses. The viscous damper is found to be effective for response control of adjacent structures by connecting with appropriate damping coefficient of damper. The optimum damping of viscous damper and corresponding response quantities are obtained for different frequency ratio and mass ratio for viscously damped connected structures. Explicit expressions for optimum damper damping are then obtained for minimum responses of the coupled structures using curve-fitting technique that can be conveniently used for applications in dynamical systems. The error in these expressions is found to be negligible. Further, numerical results had indicated that the frequency ratio affect the optimum damper damping and corresponding responses, where as mass ratio and damping of the connected structures does not have noticeable effect on the optimum responses. A simulation study has also been carried out to examine the effectiveness of the explicit expressions.**

Keywords-Adjacent Structures; Optimum Damping; Steady-state Response; Stationary Response; Viscous Damper

I. INTRODUCTION

Structural vibration control, as an advanced technology in engineering, consists of implementing energy dissipation devices into structures to reduce excessive structural vibrations to prevent catastrophic structural failure and enhance human comfort due to natural disturbances like strong wind and earthquakes. In early 1990s, considerable attention has been paid to research and development of structural control devices, and medium and high rise structures have begun implementing energy dissipation devices or control system to reduce excessive structural vibrations. These control strategies able to modify dynamically the response of the structure in a desirable manner, thereby termed protective systems for the structures.

When possible, the coupling of two adjacent structures with suitable mechanisms is a developing method among the various control techniques. The concept is to exert control forces upon one another to reduce the overall responses of the system. The free space available between two adjacent structures can be effectively utilized for placing the control devices and does not require additional space for the installation of such devices. Such type of arrangement is also prevent the mutual pounding between two adjacent structures, occurred in the past major seismic events such as 1985 mexico City earthquake [1], the 1989 Loma Prieta earthquake [2] and many others.

Connecting the adjacent structures with passive energy dissipation devices is very effective in mitigating the dynamic responses as well as minimizing the chances of pounding. Past studies [3,4,5,6,7,8] found the passive energy dissipation devices to be very effective in mitigating the dynamic responses of adjacent coupled structures as well as minimizing the chances of pounding. The present study, therefore, aims at developing explicit formulae for optimum damping coefficient of damper for damped coupled system subjected to base acceleration and response quantity being minimized. The specific objectives of the present study are (i) to obtain the optimum damping coefficient of damper for damped adjacent coupled structures subjected to base acceleration modeled as harmonic excitation as well as Gaussian white-noise random process, (ii) to study the effect of system parameter such as mass ratio frequency ratio and damping in the main system on the optimum damping of viscous damper and corresponding responses, and (iii) to derive explicit expressions for the optimum damper damping for practical application.

II. STRUCTURAL MODEL

Let us consider two adjacent structures connected with a viscous damper as shown in Figure 1(a). The adjacent structures are idealized as SDOF systems and referred as Structure 1 and 2. The two structures are assumed to be symmetric with their symmetric planes in alignment. The ground motion is assumed to occur in one direction in the symmetric planes of the structures so that the problem can be simplified as a two-dimensional problem as shown in Fig. 1(a). Both structures are assumed to be supported on stiff ground and subjected to the same ground acceleration. The viscous damper is modeled as linear dash pot, in which the force is proportional to the relative velocity of its both ends. The corresponding mathematical model of the damper connected structures is shown in Fig. 1(b).

Let m_1, c_1, k_1 and $m_2,\ c_2, k_2$ be the mass, damping coefficient and stiffness of the Structure 1 and 2, respectively. The natural frequency of the structure is given by $\omega_1 = \sqrt{k_1/m_1}$ and $\omega_2 = \sqrt{k_2/m_2}$ for Structure 1 and 2, respectively. The damping ratio of Structures 1 and 2, is given by $\xi_1 = c_1 / 2m_1\omega_1$ and $\xi_2 = c_2 / 2m_2\omega_2$, respectively.

Let β and μ be the frequency and mass ratios of two structures defined as

$$\beta = \frac{\omega_2}{\omega_1} \tag{1}$$

$$\mu = \frac{m_1}{m_2} \tag{2}$$

Let c_d be the damping coefficient of the damper, which is expressed in the normalized form as

$$\xi_d = \frac{c_d}{2m_1\omega_1} \tag{3}$$

where ξ_d is the normalized damping coefficient of damper. The governing equations of motion for the damper connected system can be written as

$$m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 + c_d(\dot{x}_1 - \dot{x}_2) = -m_1\ddot{x}_g \tag{4a}$$

$$m_2\ddot{x}_2 + c_2\dot{x}_2 + k_2x_2 - c_d(\dot{x}_1 - \dot{x}_2) = -m_2\ddot{x}_g \tag{4b}$$

Where x_2 and x_2 are the displacement responses, relative to the ground, of Structures 1 and 2, respectively; and $\ddot{x}_g$ is the ground acceleration. The structural control criteria depend on the nature of dynamic loads and the response quantities of interest. Minimizing the relative displacement or absolute acceleration of the system has always been considered as the control objective. In case of flexible structures, displacements are predominant that need to be controlled. On contrary to this, in case of stiff structures, accelerations are of more concern generating higher inertial forces in structures, which should be mitigated. In view of this, the study aims to arrive the distinct expressions for optimum parameters of damper for minimizing exclusively displacement as well as acceleration responses.

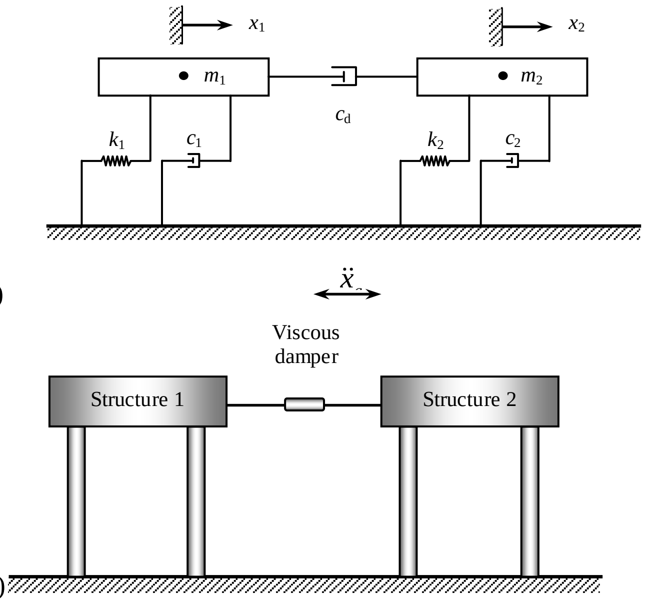

Fig. 1 Structural model of two SDOF adjacent coupled structures

III. RESPONSE TO HARMONIC EXCITATION

Let us consider the coupled system subjected to harmonic base acceleration given by

$$\ddot{x}_g = a_0 e^{i\omega t} \tag{5}$$

where a_0 and ω are the amplitude and excitation frequency, respectively of the harmonic ground motion. Thus, from Equation (4), the steady-state responses x_1 and x_2 are obtained as

$$x_1 = \frac{N_1}{D} a_0 e^{i\omega t} \text{ and } x_2 = \frac{N_2}{D} a_0 e^{i\omega t} \tag{6a,b}$$

where

$$N_1 = \omega^2 - \omega_2^2 - i\omega(\Delta_d + \Delta_2) \tag{7a}$$

$$N_2 = \omega^2 - \omega_1^2 - i\omega(\Delta_d + \Delta_1) \tag{7b}$$

$$\begin{aligned} D &= \omega^4 + (i\omega)^3(\Delta_1 + \Delta_2 + \Delta_d) \\ &+ (i\omega)^2(\omega_1^2 + \omega_2^2 + \Delta_1\Delta_2 + \Delta_1\Delta_{d2} + \Delta_2\Delta_{d1}) \\ &+ i\omega(\omega_1^2\Delta_2 + \omega_2^2\Delta_1 + \omega_1^2\Delta_{d2}\omega_2^2\Delta_{d1}) + \omega_1^2\omega_2^2 \end{aligned} \tag{8}$$

with

$$\Delta_1 = c_1/m_1 \quad \Delta_2 = c_2/m_2 \quad \Delta_{d1} = c_e/m_1 \quad \Delta_d = \Delta_{d1} + \Delta_{d2} \tag{9}$$

The absolute accelerations ($\ddot{x}_{a1}$ and $\ddot{x}_{a2}$) can be calculated by differentiating Equations (6) twice and adding it to the ground acceleration as given below

$$\ddot{x}_{a1} = \frac{N_{a1}}{D} a_0 e^{i\omega t} \text{ and } \ddot{x}_{a2} = \frac{N_{a2}}{D} a_0 e^{i\omega t} \tag{10}$$

Where

$$\begin{aligned} N_{a1} &= (i\omega)^3(\Delta_1) + (i\omega)^2(\Delta_1\Delta_2 + \Delta_1\Delta_{d2} + \Delta_2\Delta_{d1} + \omega_1^2) \\ &+ (i\omega)(\omega_1^2\Delta_2 + \omega_2^2\Delta_1 + \omega_1^2\Delta_{d2} + \omega_2^2\Delta_{d1}) + \omega_1^2\omega_2^2 \end{aligned} \tag{11a}$$

$$\begin{aligned} N_{a2} &= (i\omega)^3(\Delta_2) + (i\omega)^2(\Delta_1\Delta_2 + \Delta_1\Delta_{d2} + \Delta_2\Delta_{d1} + \omega_2^2) \\ &+ (i\omega)(\omega_1^2\Delta_2 + \omega_2^2\Delta_1 + \omega_1^2\Delta_{d2} + \omega_2^2\Delta_{d1}) + \omega_1^2\omega_2^2 \end{aligned} \tag{11b}$$

From Equation (6) the steady state amplitude of displacement responses of two structures are expressed as

$$|x_1| = \sqrt{\frac{(\omega^2 - \omega_2^2)^2 + (\omega(\Delta_d + \Delta_2))^2}{\begin{gathered}(\omega^4 - \omega^2(\omega_1^2 + \omega_2^2 + \Delta_1\Delta_2 + \Delta_1\Delta_{d2} + \Delta_2\Delta_{d1}) + \omega_1^2\omega_2^2)^2 \\ (\omega(\omega_1^2\Delta_2 + \omega_2^2\Delta_1 + \omega_1^2\Delta_{d2}\omega_2^2\Delta_{d1}) - \omega^3(\Delta_1 + \Delta_2 + \Delta_d))^2\end{gathered}}}\, a_0 \tag{12a}$$

$$|x_2| = \sqrt{\frac{(\omega^2 - \omega_1^2)^2 + (\omega(\Delta_d + \Delta_1))^2}{\begin{gathered}(\omega^4 - \omega^2(\omega_1^2 + \omega_2^2 + \Delta_1\Delta_2 + \Delta_1\Delta_{d2} + \Delta_2\Delta_{d1}) + \omega_1^2\omega_2^2)^2 \\ (\omega(\omega_1^2\Delta_2 + \omega_2^2\Delta_1 + \omega_1^2\Delta_{d2}\omega_2^2\Delta_{d1}) - \omega^3(\Delta_1 + \Delta_2 + \Delta_d))^2\end{gathered}}}\, a_0 \tag{12b}$$

and from Equation (10), the steady-state amplitude of acceleration responses of two structures are expressed as

$$|\ddot{x}_{a1}| = \sqrt{\frac{\begin{gathered}(\omega_1^2\omega_2^2 - \omega^2(\Delta_1\Delta_2 + \Delta_1\Delta_{d2} + \Delta_2\Delta_{d1} + \omega_1^2))^2 \\ (\omega(\omega_1^2\Delta_2 + \omega_2^2\Delta_1 + \omega_1^2\Delta_{d2} + \omega_2^2\Delta_{d1}) - \omega^3(\Delta_1))^2\end{gathered}}{\begin{gathered}(\omega^4 - \omega^2(\omega_1^2 + \omega_2^2 + \Delta_1\Delta_2 + \Delta_1\Delta_{d2} + \Delta_2\Delta_{d1}) + \omega_1^2\omega_2^2)^2 \\ (\omega(\omega_1^2\Delta_2 + \omega_2^2\Delta_1 + \omega_1^2\Delta_{d2}\omega_2^2\Delta_{d1}) - \omega^3(\Delta_1 + \Delta_2 + \Delta_d))^2\end{gathered}}}\, a_0 \tag{13a}$$

$$|\ddot{x}_{a2}| = \sqrt{\frac{\begin{gathered}(\omega_1^2\omega_2^2 - \omega^2(\Delta_1\Delta_2 + \Delta_1\Delta_{d2} + \Delta_2\Delta_{d1} + \omega_2^2))^2 \\ +(\omega)(\omega_1^2\Delta_2 + \omega_2^2\Delta_1 + \omega_1^2\Delta_{d2} + \omega_2^2\Delta_{d1}) - \omega)^3(\Delta_2))^2\end{gathered}}{\begin{gathered}(\omega^4 - \omega^2(\omega_1^2 + \omega_2^2 + \Delta_1\Delta_2 + \Delta_1\Delta_{d2} + \Delta_2\Delta_{d1}) + \omega_1^2\omega_2^2)^2 \\ (\omega(\omega_1^2\Delta_2 + \omega_2^2\Delta_1 + \omega_1^2\Delta_{d2}\omega_2^2\Delta_{d1}) - \omega^3(\Delta_1 + \Delta_2 + \Delta_d))^2\end{gathered}}}\, a_0 \tag{13b}$$

Considering the two SDOF connected structures with their mass ratio $\mu = 1$ and frequency ratio $\beta = 2$. Thus, the Structure 1 is said to be soft structure and Structure 2 is said to be stiff structure. The variation of displacement amplitude response and absolute acceleration amplitude response against the excitation frequency for five values of the damper damping coefficient (i.e. ξ_d =0, 0.1, 0.276, 0.447 and 1) are shown in Fig. 2 for undamped and damped connected structures with structural damping ratio $\xi_1 = \xi_2$ =0.05.

From Fig. 2, it is observed that the peak displacements of both structures are reduced up to certain value of the damper damping coefficient after which they are increased. This is because, the higher damper damping coefficient reduces the relative velocity of damper and hence the energy absorbing capacity from damping force decreases. When the damper damping coefficient is too high, the relative velocity and displacement of the two structures become nearly zero so that the two structures behave as though they are almost rigidly connected. Significant reductions can be achieved in responses of both structures when they are connected with viscous damper of optimum damping implying that the viscous dampers are quite effective in enhancing the seismic performance of connected structures. Further, from Fig. 2, it is interesting to note that in the connected undamped system, all curves are intersecting at a common point (refer point '*S*') irrespective of damping coefficients of damper and whereas, this phenomena is not seen in case of the connected damped system.

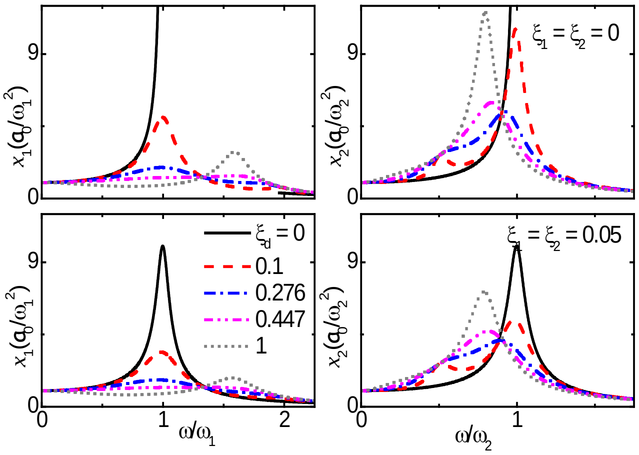

Fig. 2 Variation of displacements responses against excitation frequency ($\beta = 2$ and $\mu = 1$)

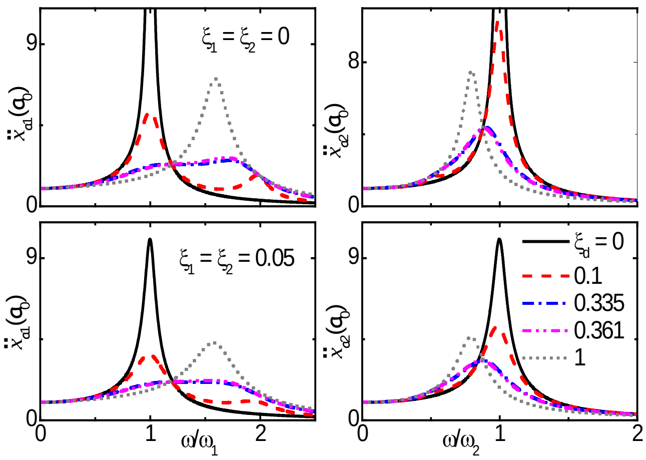

Fig. 3 Variation of absolute acceleration responses against excitation frequency ($\beta = 2$ and $\mu = 1$)

Thus, it can be concluded that connecting the adjacent structures with viscous damper enhances the seismic performance of both structures significantly and in case of connected undamped system, there exists a common point through which all curves pass irrespective of damping coefficient of damper. Similar effects of damper damping on absolute accelerations of connected structures are also observed in Fig. 3, which shows the variation of absolute accelerations, against the excitation frequency.

The variations of the peak displacement and the peak acceleration responses of two structures against the damping coefficient of damper (mass ratio $\mu = 1$ and frequency ratio $\beta = 2$) are shown in Fig. 4 for different damping ratios in connected structures (i.e. $\xi_1 = \xi_2$ =0, 0.02, and 0.05). It is clear from graphs that there exists an optimum value for the damper damping coefficient to yield minimum responses. Further, the optimum damping coefficient of damper for two structures is different, but both optimum damper damping are close to each other, and in the vicinity of it, the response of the two connected structures does not vary significantly implying that the optimum damping for one structure also reduces the response of the other structure substantially.

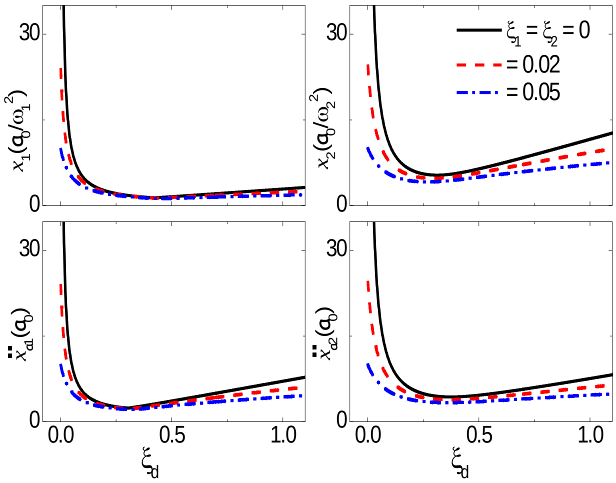

Fig. 4 Variation of peak displacement and peak absolute accelerations against damping coefficient of damper ($\beta = 2$ and $\mu = 1$)

IV. OPTIMUM DAMPER DAMPING FOR CONNECTED STRUCTURES

A. Minimization of Peak Displacement Response

In the optimal design of viscous damper connecting adjacent undamped structures subjected to harmonic excitation, fixed point "*S*" is obtained at which the response is independent of damper damping and further, the maximum steady state response occur at the fixed point. Thus, the horizontal tangent at this fixed point gives the optimum damper damping. In case of damped connected structures, there is not any fixed point but the response curves for different damper damping intersect at points clustered in the very close vicinity of the fixed point frequency, obtained for undamped coupled structures. Further, from Fig. 2, it is also observed that maximum steady state response occur at point clustered in the close vicinity of the fixed point. The values of $\xi_d = 0.276$ and 0.447 are taken from Fig. 2 which corresponds to the value of damper damping for which the maximum amplitude attains the minimum value for Structures

2 and 1, respectively, and the horizontal tangent at this point gives the optimal damper damping. In the present case, the viscous damper is connecting damped structural system ($\xi_1 = \xi_2 = \xi$), it is assumed that fixed point condition also approximately holds good even for moderate damping in the structural system, i.e. there exist fixed or invariant point on the response versus excitation frequency plot where response is almost independent of structural damping, i.e. steady state amplitude x_1 and $\ddot{x}_{a1}$ is independent of ξ_1; and x_2 and $\ddot{x}_{a2}$ is independent of ξ_2 . With this assumption, it is possible to equate the response amplitude for two extreme values of ξ_d, namely for $\xi_d = 0$ and $\xi_d = \infty$. This, solving for ω and after algebraic simplification, will give the excitation frequency for Structure 1, expressed by

$$\omega_{x1}^2 = \frac{\omega_1^2(1+\beta)(1+2\mu+\beta^2)}{2(1+\beta)(1+\mu)-4(1+2\mu+\beta)\xi^2} \tag{14a}$$

Similarly, using $\xi_d = 0$ and $\xi_d = \infty$ in Equation (12b) and equating them, solving for ω will give the excitation frequency for Structure 2, expressed by

$$\omega_{x2}^2 = \frac{\omega_2^2(1+\beta)(\mu+\beta^2(2+\mu))}{2\beta^2((1+\beta)(1+\mu)-2(\mu+\beta(2+\mu))\xi^2)} \tag{14b}$$

The optimum damping of damper for response x_1 and x_2 can be obtained by setting the slopes of the curves equal to zero at ω_{x1} and ω_{x2}, respectively. By setting the slopes of the response curves equal to zero, it gives fourth degree equation, which is quite complex and cannot be easily solved for the closed-form expressions for optimum parameter of the damper. The alternate option for optimum damper parameter is derived explicit expression. The optimum damper damping for undamped structures is very useful for obtaining an explicit expression for optimum damper damping coefficient for damped structures. Close-form expression for optimum damping coefficient of damper for displacement x_1 and x_2 of connected undamped structures given by [9] are

$$\xi_{d,ux_1}^{opt} = \left|\frac{(1-\beta^2)(1+2\mu)}{\sqrt{8(1+\mu)^3(1+2\mu+\beta^2)}}\right|$$

and

$$\xi_{d,ux_2}^{opt} = \left|\frac{(1-\beta^2)(2+\mu)}{\sqrt{8(1+\mu)^3(\mu+2\beta^2+\mu\beta^2)}}\right| \tag{15}$$

It has been observed that optimum damping coefficient increases with increase of the frequency ratio; and increase in mass ratio reduces the optimum damping coefficient of damper. From Equation (6), the optimum parameters of the damper for minimum displacement responses are calculated numerically for various values of the frequency ratio and damping in connected structures, considering constant mass ratio. Then using a curve fitting technique as explain in [10], explicit expression are arrived at for the optimum parameter of the damper in terms of the structural parameters as given below

$$\begin{aligned}\xi_{d,x_1}^{opt} &= \left|\frac{(1-\beta^2)(1+2\mu)}{(1+\mu)\sqrt{8(1+\mu)(1+2\mu+\beta^2)}}\right| \\ &+(-0.017\xi+0.424\xi^2)+(-0.476\xi+7.523\xi^2)\beta \\ &+(0.311\xi-3.293\xi^2)\beta^2\end{aligned} \tag{16a}$$

$$\begin{aligned}\xi_{d,x_2}^{opt} &= \left|\frac{(1-\beta^2)(2+\mu)}{(1+\mu)\sqrt{8(1+\mu)(\mu+2\beta^2+\mu\beta^2)}}\right| \\ &+(0.376\xi^2-28.41\xi^4) \\ &+(0.252-7.056\xi^2+46.153\xi^3)\beta \\ &+(-0.339\xi+1.966\xi^2)\beta^2\end{aligned} \tag{16b}$$

B. Minimization of Peak Acceleration Response.

From Equation (10), the optimum parameters of the damper for minimum acceleration responses are calculated numerically for various values of the frequency ratio and damping in connected structures, considering constant mass ratio. Then using a curve fitting technique, explicit equations are arrived at for the optimum parameters of the damper in terms of the structural parameters as given below

$$\begin{aligned}\xi_{d,x_{a1}}^{opt} &= \left|\frac{(\beta^2-1)(\beta^2+2\mu)}{2(\beta^2+\mu)}\sqrt{\frac{1}{2(1+\mu)(1+2\mu+\beta^2)}}\right|+(-0.019\xi+0.428\xi^2) \\ &+(1.919\xi-100.26\xi^2+1555.26\xi^3-7329.19\xi^4)\beta \\ &+(-1.358\xi+66.83\xi^2-1022.55\xi^3+4804.19\xi^4)\beta^2\end{aligned} \tag{17a}$$

$$\begin{aligned}\xi_{d,x_{a2}}^{opt} &= \left|\frac{(\beta^2-1)(2\beta^2+\mu)}{2(\beta^2+\mu)}\sqrt{\frac{1}{2(1+\mu)(\mu+2\beta^2+\mu\beta^2)}}\right| \\ &+(0.004\xi+0.128\xi^2) \\ &+(-0.102\xi+1.586\xi^2)\beta+(-0.043\xi-0.909\xi^2)\beta^2\end{aligned} \tag{17b}$$

V. RESPONSE TO STATIONARY WHITE NOISE RANDOM EXCITATION

Let the coupled system with structural damping $\xi_1 = \xi_2 = \xi$ subjected to the Gaussian white-noise with constant power spectral density S_0 . The mean square displacement ($\sigma_{x_1}^2$ and $\sigma_{x_2}^2$) of Structure1 and 2, respectively, can be expressed as

$$\sigma_{x_1}^2 = \int_{-\infty}^{\infty}|x_1(i\omega)|^2 S_0\, d\omega \tag{18a}$$

$$\sigma_{x_2}^2 = \int_{-\infty}^{\infty}|x_2(i\omega)|^2 S_0\, d\omega \tag{18b}$$

where $|x_1(i\omega)|$ and $|x_2(i\omega)|$ are amplitude of steady state displacement response of Structure 1 and 2 , respectively given by Equations (12a) and (12b), respectively. The mean square acceleration ($\sigma_{a_1}^2$ and $\sigma_{a_2}^2$) of Structure1 and 2, respectively, can be expressed as

$$\sigma_{a_1}^2 = \int_{-\infty}^{\infty} \left| \ddot{x}_{a_1}(i\omega) \right|^2 S_0 \, d\omega \tag{19a}$$

$$\sigma_{a_2}^2 = \int_{-\infty}^{\infty} \left| \ddot{x}_{a_2}(i\omega) \right|^2 S_0 \, d\omega \tag{19b}$$

where $\left|\ddot{x}_{a_1}(i\omega)\right|$ and $\left|\ddot{x}_{a_2}(i\omega)\right|$ are amplitude of steady state acceleration response of Structure 1 and 2 , respectively given by Equations (13a) and (13b), respectively.

VI. OPTIMUM DAMPER DAMPING FOR CONNECTED STRUCTURE

1) Minimization of the mean square displacement response.

The mean square displacement responses are obtained by solving the integral of Equation (18), and using the technique given in [11], can be given as

$$\sigma_{x_1}^2 = \frac{2\pi S_0}{\omega_1^3}\left[\frac{\begin{array}{l}\beta(1+\beta)^2\xi((-1+\beta)^2+4\beta\xi^2)\\ +(1+\beta)((-1+\beta)^2(1+\beta)\mu\\ +4\beta(2+\beta+\beta^2+2(1+\beta)\mu)\xi^2)\xi_e\\ +4(1+\mu)(1+\mu+\beta(3+\beta^2+(3+\beta)\mu))\xi\xi_e^2\\ +4(1+\mu)^3\xi_e^3\end{array}}{\begin{array}{l}4(\beta(1+\beta)^2\xi^2((-1+\beta)^2+4\beta\xi^2)\\ +(1+\beta)^2(\mu+\beta)\xi((-1+\beta)^2+8\beta\xi^2)\xi_e\\ \quad +((1+\beta^2)^2\mu+4(\beta^4+\mu^2+3\beta^3(1+\mu)\\ +3\beta\mu(1+\mu)+\beta^2(1+\mu(4+\mu)))\xi^2)\xi_e^2\\ \quad +4(1+\mu)(\mu+\beta)(\mu+\beta^2)\xi\xi_e^3)\end{array}}\right] \tag{20a}$$

$$\sigma_{x_2}^2 = \frac{2\pi S_0}{\omega_2^3}\left[\frac{\begin{array}{l}\beta^2(4\beta(1+\beta)^2\xi^3+\xi_e+4(1+\beta)(\mu\\ +\beta(2+\mu+2\beta(1+\mu)))\xi^2\xi_e\\ \quad +\beta^2\xi_e(-2+\beta^2+4(1+\mu)^3\xi_e^2)\\ \quad +\xi((-1+\beta^2)^2+4(1+\mu)(\mu\\ +\beta(1+\beta(3+\beta)(1+\mu)))\xi_e^2))\end{array}}{\begin{array}{l}4(\beta(1+\beta)^2\xi^2((-1+\beta)^2+4\beta\xi^2)\\ +(1+\beta)^2(\mu+\beta)\xi((-1+\beta)^2+8\beta\xi^2)\xi_e\\ \quad +((1+\beta^2)^2\mu+4(\beta^4+\mu^2+3\beta^3(1+\mu)\\ +3\beta\mu(1+\mu)+\beta^2(1+\mu(4+\mu)))\xi^2)\xi_e^2\\ \quad +4(1+\mu)(\mu+\beta)(\mu+\beta^2)\xi\xi_e^3)\end{array}}\right] \tag{20b}$$

The optimizing condition $d\sigma_{x_1}^2 \big/ d\xi_d = 0$ gives the optimum damping for the Structure1, but again it gives fourth degree equation, which is quite complex and cannot be easily solved for the closed-form expressions for optimum parameter of the damper. From Equation (20), the optimum parameters of the damper for minimum value of mean square displacement responses are calculated numerically for various values of the frequency ratio and damping in connected structures, considering constant mass ratio. Then using a curve fitting technique as explain in [10], explicit expression are arrived at for the optimum parameter of the damper in terms of the structural parameters as given below

$$\begin{aligned}\xi_{d,\sigma_{x1}^2}^{opt} = {} & \left|\frac{(\beta^2-1)}{2(1+\mu)}\sqrt{\frac{\mu}{(1+\mu)}}\right| + (0.2186\xi + 9.657\xi^2)\\ & + (-2.104\xi + 5.6699\xi^2)\beta\\ & + (0.9487\xi + 22.7944\xi^2)\beta^2\end{aligned} \tag{21a}$$

$$\begin{aligned}\xi_{d,\sigma_{x2}^2}^{opt} = {} & \left|\frac{(\beta^2-1)}{2\beta(1+\mu)}\sqrt{\frac{1}{(1+\mu)}}\right| + (0.2424\xi - 0.995\xi^2)\\ & + (-0.09846\xi - 0.4812\xi^2)\beta\\ & + (-0.2823\xi + 0.4855\xi^2)\beta^2\end{aligned} \tag{21b}$$

Minimization of the mean square acceleration response.

The mean square acceleration responses are obtained by solving the integral of Equation (19), and using the technique given in [11], can be given as

$$\sigma_{a_1}^2 = 2\pi S_0\omega_1\left[\frac{\begin{array}{l}\beta(1+\beta)^2\xi((-1+\beta)^2+4\beta\xi^2)(\xi+4\xi^3)\\ +(1+\beta)((1+\beta)\mu(1+4\xi^2)((-1+\beta)^2+8\beta\xi^2)\\ +4\beta^2\xi^2(3-\beta+2\beta^2+4(1+3\beta)\xi^2))\xi_e\\ +4\xi(3\beta^3+\beta^4+\beta^5+4\beta\mu+\beta^2\mu\\ +3\beta^3\mu+2\beta^4\mu+\mu^2+3\beta\mu^2+\beta^2\mu^2\\ +4(\beta+\mu)(\mu+\beta(3\mu+\beta(2+3\beta+\mu)))\xi^2)\xi_e^2\\ +4(\beta^2+\mu)((1+\mu)(\beta^2+\mu)+4(\beta+\mu)^2\xi^2)\xi_e^3\end{array}}{\begin{array}{l}4(\beta(1+\beta)^2\xi^2((-1+\beta)^2+4\beta\xi^2)\\ +(1+\beta)^2(\mu+\beta)\xi((-1+\beta)^2+8\beta\xi^2)\xi_e\\ +((1+\beta^2)^2\mu+4(\beta^4+\mu^2+3\beta^3(1+\mu)\\ +3\beta\mu(1+\mu)+\beta^2(1+\mu(4+\mu)))\xi^2)\xi_e^2\\ \quad +4(1+\mu)(\mu+\beta)(\mu+\beta^2)\xi\xi_e^3)\end{array}}\right] \tag{22a}$$

$$\sigma_{a_2}^2 = 2\pi S_0\omega_1\left[\frac{\begin{array}{l}\beta^2(1+\beta)^2\xi(1+4\xi^2)((-1+\beta)^2+4\beta\xi^2)\\ +\beta(1+\beta)((-1+\beta)^2\beta(1+\beta)+4(2\mu\\ +\beta(1+\beta+\beta^2+\beta^3-\mu+3\beta\mu))\xi^2\\ +16\beta(2\beta(1+\beta)+(3+\beta)\mu)\xi^4)\xi_e\\ +4\xi(\beta^3+3\beta^4+\beta^5+2\beta\mu+3\beta^2\mu\\ +\beta^3\mu+4\beta^4\mu+\mu^2+\beta\mu^2+3\beta^2\mu^2\\ +4\beta(\mu+\beta)(3\mu+\beta(1+\beta(3+\beta)+2\mu))\xi^2)\xi_e^2\\ +4(\beta^2+\mu)((1+\mu)(\beta^2+\mu)+4(\beta+\mu)^2\xi^2)\xi_e^3\end{array}}{\begin{array}{l}4(\beta(1+\beta)^2\xi^2((-1+\beta)^2+4\beta\xi^2)\\ +(1+\beta)^2(\mu+\beta)\xi((-1+\beta)^2+8\beta\xi^2)\xi_e\\ +((1+\beta^2)^2\mu+4(\beta^4+\mu^2+3\beta^3(1+\mu)\\ +3\beta\mu(1+\mu)+\beta^2(1+\mu(4+\mu)))\xi^2)\xi_e^2\\ +4(1+\mu)(\mu+\beta)(\mu+\beta^2)\xi\xi_e^3)\end{array}}\right] \tag{22b}$$

The optimizing condition $d\sigma^2_{a_1} / d\xi_d = 0$ gives the optimum damping for the Structure1, but again it gives fourth degree equation, which is quite complex and cannot be easily solved for the closed-form expressions for optimum parameter of the damper. From Equation (22), the optimum parameters of the damper for minimum value of mean square acceleration responses are calculated numerically for various values of the frequency ratio and damping in connected structures, considering constant mass ratio. Then using a curve fitting technique as explain in [10], explicit expression are arrived at for the optimum parameter of the damper in terms of the structural parameters as given below

$$\xi^{opt}_{d,\sigma^2_{a_1}} = \left| \frac{(\beta^2 - 1)}{2(\beta^2 + \mu)} \sqrt{\frac{\mu}{(1+\mu)}} \right| + (0.167\xi - 0.0595\xi^2) + (-0.06578\xi + 0.1927\xi^2)\beta + (-0.1035\xi - 0.0396\xi^2)\beta^2 \tag{23a}$$

$$\xi^{opt}_{d,\sigma^2_{a_2}} = \left| \frac{\beta(\beta^2 - 1)}{2(\beta^2 + \mu)} \sqrt{\frac{1}{(1+\mu)}} \right| + (-0.0668\xi - 3.2254\xi^2) + (0.2429\xi - 2.6654\xi^2)\beta + (-0.000028\xi + 2.6224\xi^2)\beta^2 \tag{23b}$$

The optimum damping in damper for displacement and acceleration responses are calculated from above explicit expression and corresponding responses for different mass ratio, frequency ratio and damping in a structures are tabulated in Tables 1 and 2, respectively. It is seen from the results, that maximum error in optimum displacement responses are 5.11 %, whereas maximum error in optimum acceleration responses are 1.43 %. Thus, these explicit formulae are in very good agreement with the actual values.

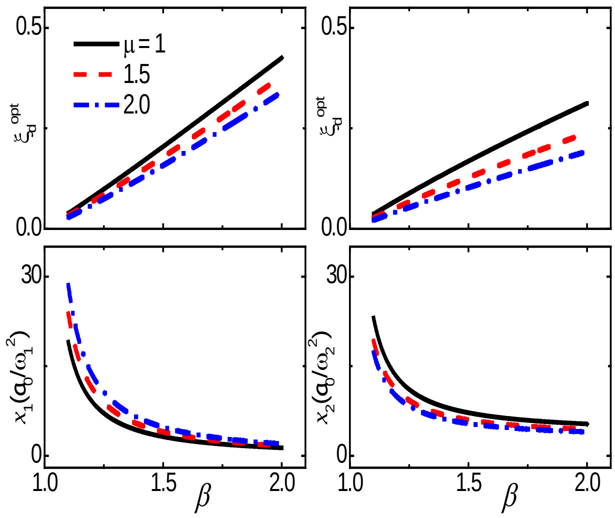

Fig. 5 Effect of frequency ration on the optimum damper damping ξ^{opt}_d and corresponding displacement responses

The variations of optimum damping of damper against frequency ratio for different mass ratio of structures and corresponding optimum responses of displacement and acceleration responses for undamped connected structures are plotted in Figures 5 and 6, respectively. It is seen from graph that the optimum damping coefficient increases with increases of the frequency ratio. This is due to the reason that higher frequency ratio increases the relative velocity between the connected floors and thus, requiring a higher damping coefficient. Further, the increase in mass ratio reduces the optimum damping coefficient of damper.

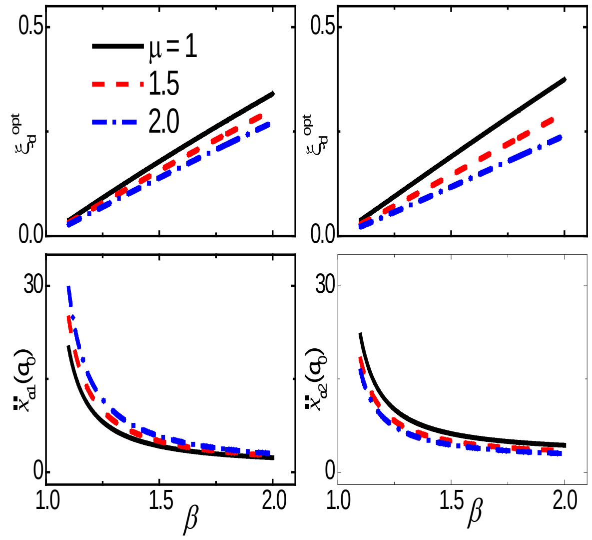

Fig. 6 Effect of frequency ration on the optimum damper damping ξ^{opt}_d and corresponding acceleration responses

The variation of optimum damping of damper against frequency ratio for different structural damping, considering mass ratio μ =1 are plotted in Figure 7. It is seen from the graph that damping in structure have negligible effect on optimum damping in damper. This is due to the reason that supplemental energy dissipation devices control the structural responses and structural damping plays marginal role for response reduction.

VII. APPLICATION TO A REAL STRUCTURE

The applicability of the above derived explicit equations for optimum damping of damper for connected structures to practical example with 20 and 10 stories, under real earthquake motion is verified. Two adjacent structures with 20 and 10 stories are considered, such that the floor mass and inter-story stiffness are assumed to be uniform for both structures. The mass and stiffness of each floor are chosen such that to yield a fundamental time period of 2.0 sec and 1.0 sec for Structures 1 and 2, respectively. The earthquake time histories selected to examine the seismic behavior of the two structures are: Imperial Valley (1940), Kobe (1995), Northridge (1994), and Loma Prieta (1989). The two structures are connected with viscous dampers at all floors and the damping in dampers is considered same in all dampers. A parametric study is carried out by varying the damping in dampers and the minimum displacement and acceleration responses of these structures under the considered earthquakes are obtained by solving the equations of motion in the incremental form using Newmark's step-by-step method assuming average acceleration over small time interval, Δt.

The response corresponding to the optimum damping of viscous damper, calculated from the explicit expression, are also obtained from the numerical procedure and are compared with the minimum responses that are obtained from the parametric study. Tables 3 and 4 present the responses for two cases, namely, (i) minimum responses obtained from

TABLE 1

EFFECT OF DAMPING IN STRUCTURES ON OPTIMUM DAMPER DAMPING FOR MINIMUM DISPLACEMENTS

μ	Damp-ing ratio	Optimum normalized damping coefficient							
		$\beta = 1.25$		1.50		1.75		2.00	
		S_1 #	S_2	S_1	S_2	S_1	S_2	S_1	S_2
1.0	0.00	0.098	0.088	0.205	0.171	0.312	0.238	0.422	0.306
	0.02	0.098	0.082	0.206	0.159	0.318	0.228	0.431	0.293
	0.05	0.104	0.071	0.214	0.143	0.328	0.208	0.445	0.266
1.5	0.00	0.085	0.067	0.178	0.127	0.277	0.184	0.379	0.235
	0.02	0.084	0.061	0.180	0.118	0.281	0.171	0.386	0.220
	0.05	0.091	0.050	0.188	0.103	0.292	0.150	0.400	0.193
2.0	0.00	0.074	0.041	0.155	0.079	0.245	0.116	0.340	0.153
	0.02	0.074	0.047	0.159	0.093	0.251	0.135	0.347	0.173
	0.05	0.080	0.036	0.167	0.077	0.261	0.113	0.360	0.146

Optimum displacements							
$\beta = 1.25$		1.50		1.75		2.00	
S_1	S_2	S_1	S_2	S_1	S_2	S_1	S_2
7.109 (0.00)*	11.090 (0.00)	3.200 (0.00)	7.196 (0.00)	1.942 (0.00)	5.936 (0.00)	1.362 (0.00)	5.332 (0.00)
5.904 (0.00)	8.569 (0.07)	0.204 (0.01)	6.062 (0.028)	1.830 (0.00)	5.150 (0.02)	1.293 (0.06)	4.690 (0.01)
4.619 (0.00)	6.304 (0.004)	2.567 (0.00)	4.878 (0.02)	1.679 (0.00)	4.283 (0.00)	1.206 (0.29)	3.965 (0.01)
8.878 (0.00)	9.255 (0.00)	3.999 (0.00)	5.998 (0.00)	2.424 (0.00)	4.948 (0.00)	1.666 (0.00)	4.444 (0.00)
6.925 (0.09)	7.609 (0.01)	3.533 (0.01)	5.277 (0.01)	2.237 (0.01)	4.453 (0.06)	1.572 (0.01)	4.042 (0.06)
5.123 (0.01)	5.930 (0.27)	2.987 (0.00)	4.454 (0.02)	1.997 (0.02)	3.863 (0.23)	1.444 (0.01)	3.558 (0.36)
10.634 (0.00)	9.310 (0.00)	4.799 (0.00)	5.898 (0.00)	2.908 (0.00)	4.784 (0.00)	2.000 (0.00)	4.246 (0.00)
7.841 (0.09)	7.501 (3.84)	4.111 (0.00)	5.318 (5.11)	2.631 (0.00)	4.463 (4.63)	1.859 (0.01)	4.013 (3.73)
5.545 (0.01)	5.760 (1.22)	3.364 (0.00)	4.239 (0.13)	2.292 (0.02)	3.732 (1.31)	1.676 (0.02)	3.467 (1.75)

S_1 - Structure 1, S_2 - Structure 2,* The value within the parenthesis indicates percentage error

TABLE 2

EFFECT OF DAMPING IN STRUCTURES ON OPTIMUM DAMPER DAMPING FOR MINIMUM ACCELERATIONS

μ	Damp-ing ratio	Optimum normalized damping coefficient								Optimum absolute accelerations							
		$\beta = 1.25$		1.50		1.75		2.00		$\beta = 1.25$		1.50		1.75		2.00	
		S_1 #	S_2	S_1	S_2	S_1	S_2	S_1	S_2	S_1	S_2	S_1	S_2	S_1	S_2	S_1	S_2
1.0	0.00	0.090	0.098	0.179	0.191	0.249	0.280	0.306	0.378	8.111 (0.00)*	10.106 (0.00)	4.200 (0.00)	6.194 (0.00)	2.955 (0.00)	4.936 (0.00)	2.386 (0.00)	4.331 (0.00)
	0.02	0.091	0.091	0.175	0.185	0.253	0.277	0.327	0.367	6.734 (0.05)	7.852 (0.06)	3.842 (0.00)	5.264 (0.03)	2.784 (0.06)	4.324 (0.00)	2.288 (1.34)	3.850 (0.01)
	0.05	0.086	0.087	0.172	0.179	0.254	0.268	0.334	0.355	5.283 (0.00)	5.846 (0.05)	3.392 (0.00)	4.300 (0.01)	2.571 (0.00)	3.654 (0.01)	2.127 (0.00)	3.312 (0.02)
1.5	0.00	0.077	0.072	0.156	0.146	0.230	0.220	0.302	0.292	9.885 (0.00)	8.257 (0.00)	4.999 (0.00)	4.998 (0.00)	3.424 (0.00)	3.948 (0.00)	2.667 (0.00)	3.444 (0.00)
	0.02	0.078	0.068	0.152	0.141	0.223	0.214	0.289	0.286	7.712 (0.08)	6.854 (0.02)	4.430 (0.03)	4.449 (0.01)	3.177 (0..08)	3.596 (0.01)	2.535 (0.18)	3.172 (0.01)
	0.05	0.074	0.064	0.151	0.135	0.224	0.205	0.295	0.275	5.741 (0.11)	5.407 (0.01)	3.774 (0.02)	3.816 (0.00)	2.862 (0.02)	3.177 (0.00)	2.352 (0.01)	2.844 (0.00)
2.0	0.00	0.072	0.050	0.141	0.116	0.209	0.179	0.272	0.241	11.646 (0.00)	7.661 (0.00)	5.798 (0.00)	4.402 (0.00)	3.909 (0.00)	3.454 (0.00)	3.000 (0.00)	3.000 (0.00)
	0.02	0.069	0.054	0.136	0.113	0.199	0.173	0.260	0.233	8.594 (0.02)	6.421 (1.43)	4.986 (0.00)	4.016 (0.00)	3.557 (0.09)	3.214 (0.00)	2.812 (0.18)	2.816 (0.00)
	0.05	0.065	0.050	0.133	0.106	0.200	0.164	0.266	0.221	6.117 (0.08)	5.158 (0.02)	4.116 (0.07)	3.546 (0.01)	3.130 (0.01)	2.913 (0.00)	2.563 (0.01)	2.586 (0.02)

S_1 - Structure 1, S_2 - Structure 2,* The value within the parenthesis indicates percentage error

parametric study, and (ii) responses corresponding to the optimum damping that obtained using explicit equations for the connected structures with damping ratio $\xi = \xi_1 = \xi_2$, 0.02 and 0.05, respectively. From the results of Tables 3 and 4, it is observed that the maximum responses corresponding to the optimum damping obtained using explicit equations are very close to the maximum responses obtained from the parametric study. Thus, it can be concluded that the derived explicit expression for optimum damping of damper are well applicable to real time problems and hence, can readily be used for preliminary design of dampers for connecting adjacent structures in practice. However, for impulsive type earthquake like Kobe (1995), there is not good match of the maximum responses corresponding to optimum damping that obtained using explicit equations and maximum responses obtained from the parametric study. Hence, for the coupled structures subjected to impulsive type excitations specific study has to be carried out for optimum damping of damper.

VIII. CONCLUSIONS

The dynamic behavior of two SDOF structures coupled by viscous damper is investigated. The viscous damper is found to be very effective in mitigating the dynamic responses of the adjacent structures and there exists an optimum damping coefficient of damper for minimum responses. The explicit expressions are obtained for optimum damping of damper to yield the minimum relative displacement and absolute accelerations under harmonic excitation and minimum mean square responses under stationary white-noise random excitation. The numerical studies shows, derived explicit expressions are in very good agreement with the actual values. The simulation study shows, that the derived explicit expression for optimum damping of damper are well applicable to real time problems and hence, can readily be used for preliminary design of dampers for connecting adjacent structures in practice.

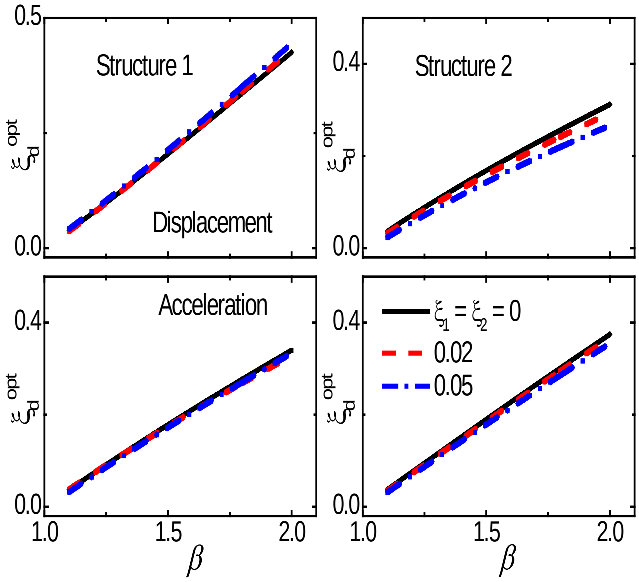

Fig. 7 Effect of frequency ration and structural damping on the optimum damping of damper

TABLE 3

APPLICABILITY OF EXPLICIT EXPRESISON TO ADJACENT 20 AND 10 STORY CONNECTED STRUCTURES UNDER EARTHQUAKE EXCITATIONS (DMAPINGRATIO IN STRUCTURE ($\xi = \xi_1 = \xi_2 = 0.02$)

Response	ξ_d^{opt}	Displacement and acceleration responses							
		Imperial Valley, 1940		Kobe, 1995		Northridge, 1994		Loma Prieta, 1989	
		I	II	I	II	I	II	I	II
x_1 (m)	0.413	0.1649	0.1658	0.2971	0.3078	0.7408	0.7431	0.7121	0.7882
x_2 (m)	0.293	0.0791	0.0807	0.1883	0.3442	0.2807	0.2811	0.2503	0.2550
$\ddot{x}_{a1}$ (m/s^2)	0.327	4.4487	4.8400	11.9607	12.7735	14.1644	15.7917	10.8827	10.967
$\ddot{x}_{a2}$ (m/s^2)	0.367	6.1484	6.2782	14.7948	15.2708	14.6790	15.1379	10.7472	10.7939

I – Minimum responses obtained from the parametric study

II – Response corresponding to the optimum damping that obtained using explicit expressions

TABLE 4

APPLICABILITY OF EXPLICIT EXPRESSION TO ADJACENT 20 AND 10 STORY CONNECTED STRUCTURE UNDER EARTHQUAKE EXCITATIONS DAMPING RATIO IN STRUCTURE ($\xi = \xi_1 = \xi_2 = 0.05$)

Response	ξ_d^{opt}	Displacement and acceleration responses							
		Imperial Valley, 1940		Kobe, 1995		Northridge, 1994		Loma Prieta, 1989	
		I	II	I	II	I	II	I	II
x_1 (m)	0.445	0.1518	0.1519	0.2656	0.2884	0.686	0.688	0.6429	0.7081
x_2 (m)	0.226	0.0742	0.0764	0.2437	0.2951	0.2624	0.2639	0.2340	0.2350
$\ddot{x}_{a1}$ (m/s^2)	0.334	3.6282	3.6378	9.7683	10.9678	12.4414	13.6437	8.9063	9.1173
$\ddot{x}_{a2}$ (m/s^2)	0.355	4.9083	5.0306	11.8719	13.7659	13.3083	13.5934	8.7323	8.7410

I – Minimum responses obtained from the parametric study

II – Response corresponding to the optimum damping that obtained using explicit expressions

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