Network Dimensioning Games in Competitive Business Environments

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***Abstract*- This paper presents a game-theoretical framework for network capacity planning strategies in competitive business environments. For this purpose, two operators are assumed to provide communication channels to a large population of users for the same price. Users will then play a game in which they try to forward their requests to the operator that will give them the lowest blocking probability. The paper discusses the equilibria resulting from some distinct user strategies. The traffic partition resulting from the user's game is an input to the operators game, in which operators try do dimension their network capacities in order to maximize their profits. In the presence of asymmetries between the channel deployment costs of both operators, we show that the operator's game will not lead to an equilibrium between pure dimensioning strategies, resulting in looping instabilities in settings where the players alternate their profit-maximizing moves. In this case, the model provides the cost asymmetry needed for an entrant to challenge the market dominance of an incumbent operator.**

Keywords-* *Telecommunication Networks; Network Dimensioning; Competitive Dynamics; Game-theory; Blocking Probability; Markovian Model; Nash Equilibrium

I. INTRODUCTION

Competition in telecommunication networks raises the need to devise pricing and dimensioning strategies with the aim of enhancing profitability in uncertain environments [1]. Such questions were ignored in the classical approach to dimension a network, which consisted of deploying a barely sufficient number of channels to assure a small enough probability of a call being blocked [2] in old telephone networks.

Clearly, the classic approach assumed that a single channel provider (operator) was allowed to operate, thus precluding any competition. If more than one operator were allowed to operate, no single operator could be held responsible for the probability of a user being blocked by all operators, hereby called the hard blocking probability. If users are free to request a channel from any operator, then both the hard blocking probability and the blocking probabilities of each operator would depend only on the numbers of channels made available to the users by all operators.

Given the competitive business environment of current networks, a new approach is now needed to discuss the dimensioning of networks that share a physical infrastructure but compete for customers in the provisioning of bandwidth to a user population under uniform pricing. In this paper, we propose a game-theoretical approach to this problem. The underlying games are played by users that wish to minimize their blocking probabilities and operators that wish to maximize their profits.

In [3], we have shown that profit maximization may be compatible with acceptable blocking performance for high enough traffic intensities, even under a monopolistic setting. We now discuss a situation in which two operators provide channels on the same link to a large population of users. Any new request for a channel will always be submitted by the user to its primary operator. Whenever such request is blocked by the primary operator, it will then be submitted by the user to another (secondary) operator. For this reason, blocking of a request by a primary operator is hereby called soft blocking.

Given the total traffic generated by the user population, the hard blocking probability depends only on the sum of the channels provided by all operators. Therefore, all other service attributes (e.g. pricing) being the same, users are likely to choose a primary operator with minimal soft blocking probability. We assume that users play a game in which they occasionally switch their primary operator in search of lower blocking probabilities of their future requests.

In Section II, we introduce a Markovian model that yields the soft blocking probabilities of two operators of a duopoly, given their numbers of deployed channels and the intensities of the primary traffic bound to each one. In Section III, we let the primary traffic intensities float under a users game until they reach equilibrium, thus obtaining the primary traffic partition between the operators under such game. Two distinct strategies are considered for the users, yielding two distinct equilibria. Based on the resulting traffic partitions, Section IV analyzes a channel capacity dimensioning game between two profit-seeking operators and variations thereof that may arise from a motivation to stabilize the operators game. Finally, Section V ends the paper with concluding remarks.

II. A SOFT BLOCKING MODEL

In [5], we have proposed a Markovian model that captures the behaviors described above and yields the resulting soft blocking probabilities of each operator when the intensities of the primary traffic bound to each one are given. Let us consider the case of a duopoly run by Operators 1 and 2. Let ν_m be the traffic intensity of the primary requests addressed to operator m and let C_m be the number of channels deployed by operator m. In a duopoly, $m \in \{1,2\}$, so the total traffic intensity generated by the users is given by:

$$\nu = \nu_1 + \nu_2 \quad , \tag{1}$$

and the total number of channels made available by all operators is:

$$C = C_1 + C_2 . \tag{2}$$

Fig. 1 illustrates the proposed Markov chain for a duopoly when $(C_1, C_2) = (3,2)$. The system is in state (i,j) when Operator 1 has $i \le C_1$ active channels and Operator 2 has $j \le C_2$ active channels. Notice that the transition rate from state (i,j) to state $(i+1,j)$ is given by ν_1 only when Operator 2 is not in a blocking state $(j < C_2)$, and by $\nu = \nu_1 + \nu_2$ when Operator 2 is in a blocking state $(j = C_2)$, so that its primary traffic ν_2 is forwarded to Operator 1. Analogously, the transition rate from state (i,j) to state $(i,j+1)$ is given by ν_2 when Operator 1 is not in a blocking state $(i < C_1)$ and by ν when Operator 1 is in a blocking state $(i = C_1)$. The downward transition rate from (i,j) to $(i-1,j)$ is given by i for any positive i, and from (i,j) to $(i,j-1)$ by j for any positive j, reflecting the standard assumption that all services are independent, exponentially distributed processes with unit mean, so that traffic rates are expressed in Erlang. All remaining transition rates are zero because all requests are assumed to demand single channels.

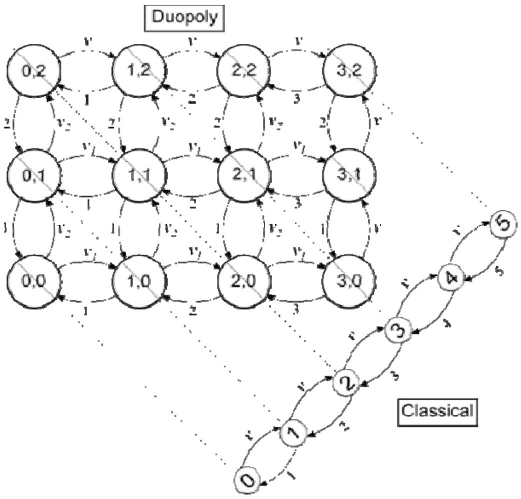

Fig. 1 Markov blocking model for a duopoly, and classical blocking model

The steady-state probability $p_{i,j}$ of each state (i,j) of the system may then be obtained from standard Markovian analysis [4]. If P_{bi} is the soft blocking probability of operator i, the soft blocking probabilities of Operators 1 and 2 may be expressed as:

$$P_{b1} = \sum_{j=0}^{C_2} p_{C_1,j} \quad (3)$$

$$P_{b2} = \sum_{i=0}^{C_1} p_{i,C_2} \quad (4)$$

Hard blocking will occur whenever a request finds the system in state (C_1, C_2) . Therefore, the hard blocking probability is given by:

$$P_b = p_{C_1,C_2} \quad (5)$$

Let U_k be the set of all states where $i + j = k$. Then, it can be seen from Fig. 1 that the sum of all transition rates from any state in U_k to all states of U_{k+1} is ν, and the sum of all rates from any state in U_k to all states of U_{k-1} is k. Therefore, a Markov process is obtained when all states in each set U_k are lumped into one state as shown in the classical model on Fig. 1.

This process yields the hard blocking probability in the form of the classical Erlang-B equation:

$$P_b = prob(k = C) = p_{C_1,C_2} = \frac{\nu^C/_{C!}}{\sum_{k=0}^{C} \nu^k/_{k!}} \quad (6).$$

III. USERS GAMES

Section II may describe a situation in which the operators are allocated to the users by a third party such as a broker. However, if users are free to choose their primary operators, the primary traffic partition (ν_1, ν_2) will result from the outcome of a game played by them. Hypothetically, let us assume that users have somehow access to information about the soft blocking probability of each operator or are willing to estimate its value by occasionally testing their secondary operator. Reliable estimates may require many requests, and thus a long time to be obtained, especially if the target blocking probability is low. After looking up this information or performing this test, the user will switch his primary operator if and only if he finds that the other (secondary) operator has a lower soft blocking probability.

In Subsections III.1 and III.2, we discuss equilibrium conditions generated by two non-cooperative games, in which each user decides independently to remain with his current primary operator or switch to another one. In each case, a specified uniform user strategy is assumed for all users. In Subsection III.3, we discuss a hypothetical situation in which all users cooperate to minimize the global rate of soft blockings.

A. A Nash Equilibrium.

Using the model introduced in Section II, the soft blocking probabilities of the two operators were calculated as a function of ν_1 when $(C_1, C_2) = (5,3)$ and $\nu = 8$, and are shown on Fig. 2. Notice that the two curves cross at a point where $P_{b1} = P_{b2}$. At the left of this point, $P_{b1} < P_{b2}$, so a primary customer of Operator 2, when testing Operator 1, would find a lower soft blocking probability and would switch his primary operator. This move would increase the primary traffic ν_1 bound to Operator 1, so the system operating point would move slightly to the right, thus approaching the crossing point. Analogously, customers of Operator 1 would switch to operator 2 whenever the system operates at the right of the crossing point, thus moving the operating point leftwards and closer to the crossing point. We conclude that the crossing point yields an algorithmic equilibrium where $P_{b1} = P_{b2}$, so that no user would be motivated to switch his primary operator for the purpose of minimizing his soft blocking probability. In game-theory parlance, this situation defines Nash equilibrium.

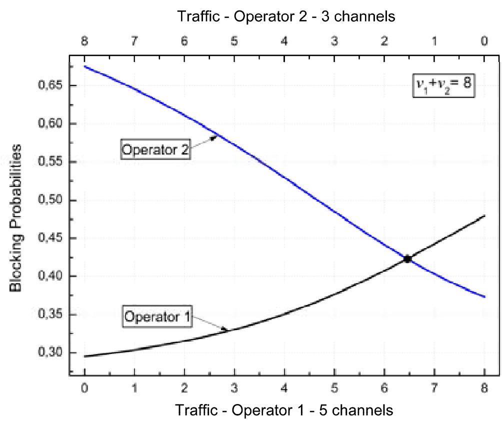

Fig. 2 Soft blocking probabilities when $(C_1, C_2) = (5,3)$.

Fig. 3 shows the soft blocking probabilities of the two operators as a function of ν_1 when $(C_1, C_2) = (7,1)$ and $\nu = 8$. In this case, the two curves do not cross within the feasible range of partitions. For any $\nu_1 \in [0,8)$, primary users of Operator 2 would switch to Operator 1 whenever testing their soft blocking probabilities. Therefore, Nash equilibrium occurs for $(\nu_1, \nu_2) = (8,0)$, with $P_{b1} < P_{b2}$.

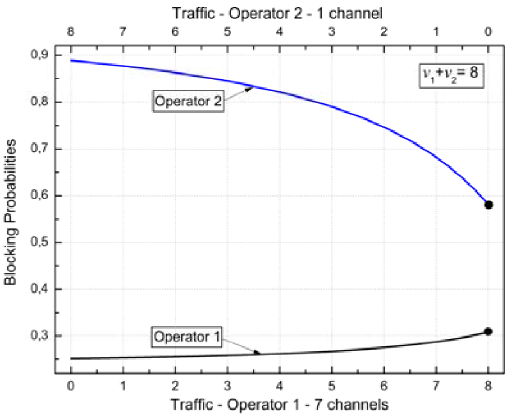

Fig. 3 Soft blocking probabilities when $(C_1, C_2) = (7,1)$.

B. An Equilibrium Generated by Impatient User Behavior

Information about the soft blocking probabilities of the operators is not likely to be available for consultation. Moreover, reliable estimation of these parameters may take a long time, prompting the users to switch operators on the basis of partial information. Taking an extreme behavior for comparison, let us consider the case of the impatient user, who switches operators whenever having a request blocked by his current primary operator and accommodated by the other operator, which then becomes the new primary operator. If all users are impatient, then the rate of user switching events from Operator 1 to 2 is given by $\sigma_1 = \nu_1(P_{b1} - P_b)$, while the rate of user switching events from Operator 2 to 1 is given by $\sigma_2 = \nu_2(P_{b2} - P_b)$. Equilibrium will emerge when $\sigma_1 = \sigma_2$, or:

$$\nu_1(P_{b1} - P_b) = \nu_2(P_{b2} - P_b) \quad (7)$$

Figs. 4 and 5 show the variations of $\sigma_1$and σ_2 with ν_1 for the same two cases discussed for Nash equilibrium in subsection III.1. Notice that the two curves will always cross now, and the crossing point defines the equilibrium generated by impatient users. This means that the operator with the smaller number of channels will always capture some primary traffic, but will offer a higher soft blocking probability. The new equilibrium is not fair in the short term because some users will feel a higher soft blocking probability than others even in cases where Nash equilibrium would make all users have the same soft blocking probability. However, user impatience would still be fair in the long term, since all users would stay with each operator for some time, with the same mean sojourn time at each one for all users.

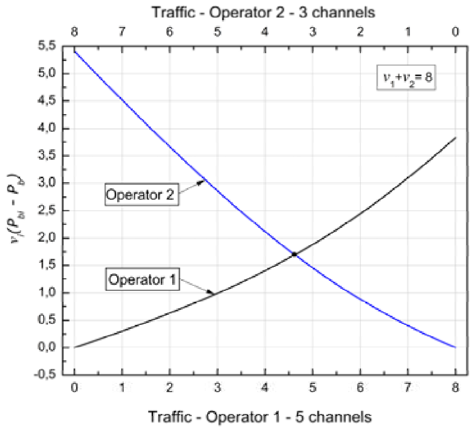

Fig. 4 Rates of impatient user switching events when $(C_1, C_2) = (5,3)$.

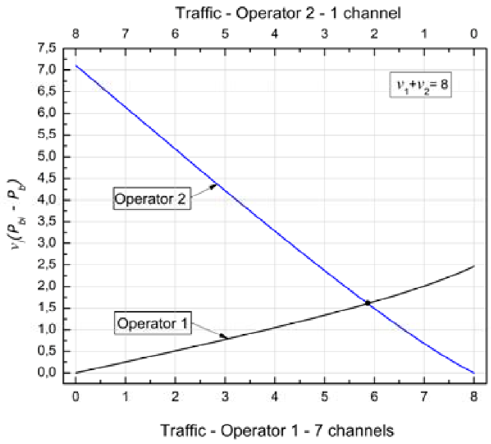

Fig. 5 Rates of impatient user switching events when $(C_1, C_2) = (7,1)$.

C. Cooperative Welfare Maximization.

The users might also be collectively interested in cooperating to produce an "efficient" traffic partition with minimal aggregate rate of soft blockings. The total rate of soft blockings is:

$$\sigma = \sigma_1 + \sigma_2 = \nu_1 P_{b1} + \nu_2 P_{b2} - \nu P_b \quad (8)$$

Given the total traffic ν generated by all users, the total rate of hard blockings νP_b is invariant with respect to ν_1. Therefore, efficiency is produced by the minimization of the total rate of primary blockings:

$$\pi = \nu_1 P_{b1} + \nu_2 P_{b2} \quad (9)$$

Figs. 6 and 7 plots the variations of π with ν_1 for the same two cases discussed in Subsections III.1 and III.2. The minimal value of π is marked with a full dot. Empty dots mark the operation points of the equilibria generated by the two non-cooperative games discussed in previous subsections. When $(C_1, C_2) = (7,1)$, Nash equilibrium is efficient and impatient user behavior is not. If $(C_1, C_2) = (5,3)$, however, the most efficient partition is somewhere in between the partitions generated by the two user behaviors discussed above. This suggests that there might be some moderately impatient user behavior resulting in the efficient partition in this case. Alternatively, efficiency may be obtained by enforcing the efficient partition. This could be done by submitting all requests to a regulating broker, who would assign each request independently to Operator 1 with probability $(\frac{\nu_{1eff}}{\nu})$ and to Operator 2 with probability $(1-\frac{\nu_{1eff}}{\nu})$, where ν_{1eff} is the value of $\nu_1$that minimizes π.

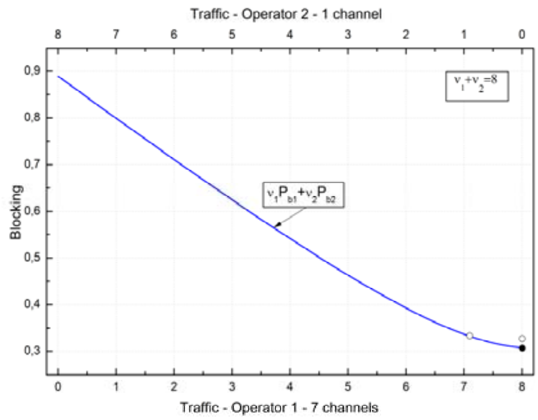

Fig. 6 Total rate of primary blockings when $(C_1, C_2) = (7,1)$.

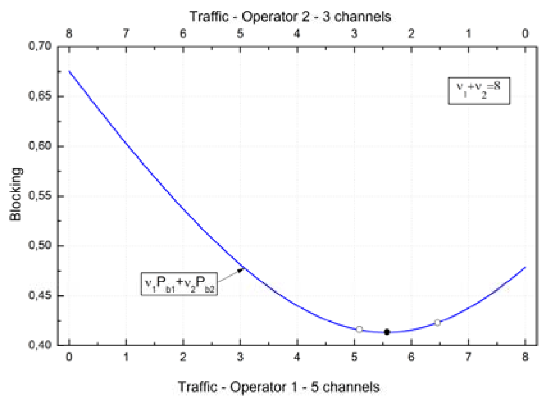

Fig. 7 Total rate of primary blockings when $(C_1, C_2) = (5,3)$.

IV. OPERATORS GAMES

In [6], the operators' game was discussed for the situation when their channel deployment costs are symmetric. In this paper, we extend these results to the case where deployment costs are asymmetric. The game-theoretical model is tested to determine the degree of cost asymmetry needed to produce plausible equilibria between two operators, thus challenging the notion of a natural monopoly. The results provide insight on the ability of innovative entrants to challenge the market dominance of an incumbent.

We normalize all revenues and costs with respect to the revenue generated per unit time by any active channel, which is then taken to be 1. The total cost incurred by operator m to deploy a channel per unit time is given by $s_m < 1$. All channels deployed by the same operator are assumed to generate the same cost, but only active channels generate revenue at any given time. When the system is in state (i, j), the instantaneous profit rates of Operators 1 and 2 are then given respectively by:

$$T_1 = i - s_1.C_1 \tag{10}$$

$$T_2 = j - s_2.C_2 \tag{11}$$

The mean profit rates, or profitabilities, of Operators 1 and 2 are then given respectively by:

$$R_1 = E(i) - s_1 C_1 \tag{12}$$

$$R_2 = E(j) - s_2 C_2 \tag{13}$$

From Little's Law [7], we know that the mean number of ongoing services in Operator 1 is its total rate (in Erlang) of incoming traffic. From an inspection of Fig. 1, we then get:

$$E(i) = \nu_1(1 - P_{b1}) + \nu_2(P_{b2} - P_b) \tag{14}$$

Likewise for Operator 2:

$$E(j) = \nu_2(1 - P_{b2}) + \nu_1(P_{b1} - P_b) \tag{15}$$

The profitabilities of Operators 1 and 2 may then be expressed as:

$$R_1 = \nu_1(1 - P_{b1}) + \nu_2(P_{b2} - P_b) - s_1 C_1 \tag{16}$$

$$R_2 = \nu_2(1 - P_{b2}) + \nu_1(P_{b1} - P_b) - s_2 C_2 \tag{17}$$

We consider a game in which each operator m chooses a number C_m of channels to be deployed with the purpose of maximizing its profitability, or payoff, R_m. For each strategy profile (C_1, C_2), the aggregate traffic ν generated by the user population will face a hard blocking probability $P_b$determined by $C = C_1 + C_2$ as in Eq. 6. A users game will determine the primary traffic partition (ν_1, ν_2) and the soft blocking probabilities P_{b1} and P_{b2} , so the profitabilities may be calculated from Eqs. 16 and 17.

Table 1 shows the normal form of the operators game when $\nu = 8$, $s_1 = s_2 = .2$ (symmetric costs), and users are free to minimize their individual soft blocking probabilities in a non-cooperative game, thus producing a Nash equilibrium between users primarily bound to each operator. Rows are indexed by C_1 and columns are indexed by C_2. Each cell shows the corresponding value of (R_1, R_2). For the sake of readability, the full accuracy of the numbers is not shown. A Nash equilibrium between operators is said to be generated by a pure strategy profile (C_1, C_2) if the corresponding cell in the normal form shows a value of R_1 that is maximal in its column and a value of R_2 that is maximal in its row. Under this condition, no operator will be motivated to change his dimensioning strategy if he believes his opponent will not change his either. In Table 1, a Nash equilibrium emerges in cell (11, 11) with profit profile (0.3, 0.3).

TABLE 1 THE NORMAL FORM OF THE OPERATORS GAME when $\nu = 8$, $s_1 = s_2 = .2$

Column	**0**		**1**		**2**		**3**		**4**		**5**		**6**		**7**		**8**		**9**		**10**		**11**		**12**		**13**		**14**	
Line	**Op .1**	**Op .2**	**Op .1**	**Op .2**	**Op .1**	**Op .2**	**Op .1**	**Op .2**	**Op .1**	**Op .2**	**Op .1**	**Op .2**	**Op .1**	**Op .2**	**Op .1**	**Op .2**	**Op .1**	**Op .2**	**Op .1**	**Op .2**	**Op .1**	**Op .2**	**Op .1**	**Op .2**	**Op .1**	**Op .2**	**Op .1**	**Op .2**	**Op .1**	**Op. 2**
0	0.0	0.0	0.0	0.6	0.0	1.2	0.0	1.8	0.0	2.2	0.0	2.6	0.0	2.8	0.0	3.0	0.0	3.0	0.0	3.0	0.0	2.9	0.0	2.8	0.0	2.6	0.0	2.4	0.0	2.2
1	0.6	0.0	0.6	0.6	0.5	1.2	0.5	1.8	0.4	2.2	0.3	2.6	0.2	2.8	0.1	3.0	0.0	3.0	-0.1	3.0	-0.1	2.9	-0.2	2.8	-0.2	2.6	-0.2	2.4	-0.2	2.2
2	1.2	0.0	1.2	0.5	1.1	1.1	0.9	1.7	0.7	2.1	0.5	2.5	0.3	2.8	0.1	2.9	-0.1	3.0	-0.2	2.9	-0.3	2.8	-0.3	2.7	-0.3	2.5	-0.4	2.4	-0.4	2.2
3	1.8	0.0	1.8	0.5	1.7	0.9	1.4	1.4	1.2	1.8	0.9	2.2	0.6	2.4	0.4	2.5	0.2	2.6	0.0	2.6	-0.2	2.6	-0.3	2.5	-0.4	2.4	-0.4	2.2	-0.5	2.1
4	2.2	0.0	2.2	0.4	2.1	0.7	1.8	1.2	1.5	1.5	1.2	1.8	0.9	2.0	0.6	2.1	0.4	2.2	0.2	2.2	0.0	2.2	-0.2	2.2	-0.3	2.1	-0.4	2.0	-0.5	1.9
5	2.6	0.0	2.6	0.3	2.5	0.5	2.2	0.9	1.8	1.2	1.5	1.5	1.1	1.6	0.8	1.8	0.5	1.9	0.3	1.9	0.1	1.9	-0.1	1.9	-0.2	1.8	-0.4	1.8	-0.5	1.7
6	2.8	0.0	2.8	0.2	2.8	0.3	2.4	0.6	2.0	0.9	1.6	1.1	1.3	1.3	1.0	1.4	0.7	1.5	0.4	1.6	0.2	1.6	0.0	1.6	-0.2	1.6	-0.3	1.5	-0.5	1.5
7	3.0	0.0	3.0	0.1	2.9	0.1	2.5	0.4	2.1	0.6	1.8	0.8	1.4	1.0	1.1	1.1	0.8	1.2	0.5	1.3	0.3	1.3	0.1	1.3	-0.1	1.3	-0.3	1.3	-0.4	1.2
8	3.0	0.0	3.0	0.0	3.0	-0.1	2.6	0.2	2.2	0.4	1.9	0.5	1.5	0.7	1.2	0.8	0.9	0.9	0.6	1.0	0.4	1.0	0.2	1.0	0.0	1.0	-0.2	1.0	-0.4	1.0
9	3.0	0.0	3.0	-0.1	2.9	-0.2	2.6	0.0	2.2	0.1	1.9	0.3	1.6	0.4	1.3	0.5	1.0	0.6	0.7	0.7	0.5	0.7	0.2	0.8	0.0	0.8	-0.2	0.8	-0.4	0.8
10	2.9	0.0	2.9	-0.1	2.8	-0.3	2.6	-0.2	2.2	0.0	1.9	0.1	1.6	0.2	1.3	0.3	1.0	0.4	0.7	0.5	0.5	0.5	0.3	0.5	0.1	0.5	-0.1	0.5	-0.3	0.5
11	2.8	0.0	2.8	-0.2	2.7	-0.3	2.5	-0.3	2.2	-0.2	1.9	-0.1	1.6	0.0	1.3	0.1	1.0	0.2	0.8	0.2	0.5	0.3	0.3	0.3	0.1	0.3	-0.1	0.3	-0.3	0.3
12	2.6	0.0	2.6	-0.2	2.5	-0.3	2.4	-0.4	2.1	-0.3	1.8	-0.2	1.6	-0.2	1.3	-0.1	1.0	0.0	0.8	0.0	0.5	0.1	0.3	0.1	0.1	0.1	-0.1	0.1	-0.3	0.1
13	2.4	0.0	2.4	-0.2	2.4	-0.4	2.2	-0.4	2.0	-0.4	1.8	-0.4	1.5	-0.3	1.3	-0.3	1.0	-0.2	0.8	-0.2	0.5	-0.1	0.3	-0.1	0.1	-0.1	-0.1	-0.1	-0.3	-0.1
14	2.2	0.0	2.2	-0.2	2.2	-0.4	2.1	-0.5	1.9	-0.5	1.7	-0.5	1.5	-0.5	1.2	-0.4	1.0	-0.4	0.8	-0.4	0.5	-0.3	0.3	-0.3	0.1	-0.3	-0.1	-0.3	-0.3	-0.3

TABLE 2 THE NORMAL FORM OF THE OPERATORS GAME WHEN $\nu = 8,\ s_1 = .2,\ s_2 = .18$

Column	0		2		4		6		8		9		10		12		14		16		18	
Line	**Op.1**	**Op.2**	**Op.1**	**Op.2**	**Op.1**	**Op.2**	**Op.1**	**Op.2**	**Op.1**	**Op.2**	**Op.1**	**Op.2**	**Op.1**	**Op.2**	**Op.1**	**Op.2**	**Op.1**	**Op.2**	**Op.1**	**Op.2**	**Op.1**	**Op.2**
0	0	0	0	1.3	0	2.4	0	3.1	0	3.4	0	3.5	0	3.4	0	3.2	0	2.9	0	2.6	0	2.3
2	1.2	0	1.1	1.2	0.7	2.3	0.3	3.1	-0.1	3.4	-0.2	3.4	-0.3	3.3	-0.3	3.1	-0.4	2.9	-0.4	2.6	-0.4	2.3
4	2.2	0	2.1	0.8	1.5	1.7	0.9	2.3	0.4	2.6	0.2	2.7	0	2.7	-0.3	2.7	-0.5	2.6	-0.6	2.4	-0.7	2.2
6	2.8	0	2.8	0.4	2	1.1	1.3	1.6	0.7	1.9	0.4	2	0.2	2.1	-0.2	2.2	-0.5	2.2	-0.7	2.1	-0.9	2
8	3	0	3	0	2.2	0.6	1.5	1	0.9	1.3	0.6	1.4	0.4	1.5	0	1.6	-0.4	1.7	-0.7	1.7	-0.9	1.6
10	2.9	0	2.8	-0.2	2.2	0.2	1.6	0.5	1	0.8	0.7	0.9	0.5	1	0.1	1.1	-0.3	1.2	-0.7	1.3	-0.9	1.2
12	2.6	0	2.5	-0.2	2.1	-0.1	1.6	0.1	1	0.4	0.8	0.5	0.5	0.6	0.1	0.7	-0.3	0.8	-0.7	0.9	-1	0.9
14	2.2	0	2.2	-0.3	1.9	-0.3	1.5	-0.2	1	0	0.8	0.1	0.5	0.2	0.1	0.3	-0.3	0.4	-0.7	0.5	-1	0.5
16	1.8	0	1.8	-0.3	1.6	-0.4	1.3	-0.4	0.9	-0.3	0.7	-0.2	0.5	-0.2	0.1	-0.1	-0.3	0	-0.7	0.1	-1	0.1
18	1.4	0	1.4	-0.3	1.3	-0.5	1.1	-0.6	0.7	-0.5	0.5	-0.5	0.3	-0.4	0	-0.4	-0.4	-0.3	-0.8	-0.2	-1.1	-0.2

Table 2 shows the same normal form when $s_2 = 0.9 s_1 = .18$, but with a larger infrastructure. For this reason, odd rows and columns are omitted in order to keep a manageable table size. No cell satisfies the condition for Nash equilibrium between pure strategies in this form. What will happen in real life will depend on the players' ability, but a hint may be obtained by first assuming a hypothetical game in which players alternate moves in which each one maximizes his/her profit after looking at Table 2. Let the game start at cell (0, 0), and let player 1 have the first move. He will deploy 8 channels, moving the system to cell (8,0), where he enjoys a maximal-profit monopoly. Operator 2 then enters the game with a slightly smaller (10%) channel deployment cost and deploys 16 channels, taking the network to cell (8, 16) where her profit is maximized under current conditions. Operator 1 will then look at Table 2 and see that he is incurring a loss, and the only way for him to minimize his loss (i.e. maximize his "profit") is to leave the game, leading the system to cell (0, 16), where Operator 2 enjoys a monopoly.

If Operator 2 keeps looking for maximal profit, she would then reduce the dimension of her network to 9 channels, thus raising her profit to 3.5. This would prompt operator 1 to re-enter the game with 12 channels, and Operator 2 to raise her number of channels to 18, thus forcing Operator 1 to leave the game again. So, if each operator makes his/her next move in search of maximal profit, the game will enter the loop given by $(0,18) \to (0,9) \to (12,9) \to (12,18) \to (0,18) \to \cdots$. However, such players' behavior would be short-sighted, since it only aims at maximal profit immediately after the next move. Eventually, next time the system goes through cell (0, 18), Operator 2 will observe that if she reduces her number of channels to 14 instead of 9, Operator 1 will not have any incentive to re-enter the game, and she can enjoy a stable, competitive monopoly, although with profit 2.9 instead of 3.5: a small price (17%) to pay for stability. On the other hand, this is still a weak stability, since it is based on the short-sightedness of Operator 1. A close examination of row 18 of Table 2 will tell Operator 1 that he may also force Operator 2 to leave the game by deploying 18 channels, if she cannot stand a lossy operation. However, this would let him with only 1.4 of profit, while the same strategy would yield profit 2.9 to Operator 2. Both players are able to hurt each other. Each can achieve a competitive monopoly if the other one is not willing to withstand losses.

TABLE 3 THE NORMAL FORM OF THE OPERATORS GAME when $\nu = 8,\ s_1 = .2,\ s_2 = .15$

Column	0		2		4		6		8		10		12		14		16		18		20		22		24		26	
Line	**Op.1**	**Op.2**	**Op.1**	**Op.2**	**Op.1**	**Op.2**	**Op.1**	**Op.2**	**Op.1**	**Op.2**	**Op.1**	**Op.2**	**Op.1**	**Op.2**	**Op.1**	**Op.2**	**Op.1**	**Op.2**	**Op.1**	**Op.2**	**Op.1**	**Op.2**	**Op.1**	**Op.2**	**Op.1**	**Op.2**	**Op.1**	**Op.2**
0	0	0	0	1.4	0	2.6	0	3.4	0	3.8	0	3.9	0	3.8	0	3.6	0	3.4	0	3.2	0	3	0	2.8	0	2.6	0	2.4
2	1.2	0	1.1	1.3	0.7	2.5	0.3	3.4	-0.1	3.8	-0.3	3.8	-0.3	3.7	-0.4	3.6	-0.4	3.4	-0.4	3.2	-0.4	3	-0.4	2.8	-0.4	2.6	-0.4	2.4
4	2.2	0	2.1	0.9	1.5	1.9	0.9	2.6	0.4	3	0	3.2	-0.3	3.3	-0.5	3.3	-0.6	3.2	-0.7	3.1	-0.8	3	-0.8	2.8	-0.8	2.6	-0.8	2.4
6	2.8	0	2.8	0.5	2	1.3	1.3	1.9	0.7	2.3	0.2	2.6	-0.2	2.8	-0.5	2.9	-0.7	2.9	-0.9	2.9	-1	2.8	-1.1	2.7	-1.1	2.5	-1.1	2.3
8	3	0	3	0.1	2.2	0.8	1.5	1.3	0.9	1.7	0.4	2	0	2.2	-0.4	2.4	-0.7	2.5	-0.9	2.5	-1.1	2.5	-1.2	2.4	-1.4	2.4	-1.4	2.2
10	2.9	0	2.8	-0.1	2.2	0.4	1.6	0.8	1	1.2	0.5	1.5	0.1	1.7	-0.3	1.9	-0.7	2.1	-0.9	2.1	-1.2	2.2	-1.4	2.2	-1.5	2.1	-1.6	2
12	2.6	0	2.5	-0.1	2.1	0.1	1.6	0.4	1	0.8	0.5	1.1	0.1	1.3	-0.3	1.5	-0.7	1.7	-1	1.8	-1.2	1.8	-1.5	1.9	-1.7	1.9	-1.8	1.8
14	2.2	0	2.2	-0.2	1.9	-0.1	1.5	0.1	1	0.4	0.5	0.7	0.1	0.9	-0.3	1.1	-0.7	1.3	-1	1.4	-1.3	1.5	-1.5	1.5	-1.8	1.6	-2	1.6
16	1.8	0	1.8	-0.2	1.6	-0.2	1.3	-0.1	0.9	0.1	0.5	0.3	0.1	0.5	-0.3	0.7	-0.7	0.9	-1	1	-1.3	1.1	-1.6	1.2	-1.9	1.3	-2.1	1.3
18	1.4	0	1.4	-0.2	1.3	-0.3	1.1	-0.3	0.7	-0.1	0.3	0.1	0	0.2	-0.4	0.4	-0.8	0.6	-1.1	0.7	-1.4	0.8	-1.7	0.9	-2	1	-2.2	1
20	1	0	1	-0.2	1	-0.4	0.8	-0.4	0.5	-0.3	0.2	-0.2	-0.2	0	-0.5	0.1	-0.9	0.3	-1.2	0.4	-1.5	0.5	-1.8	0.6	-2.1	0.7	-2.3	0.7
22	0.6	0	0.6	-0.2	0.6	-0.4	0.5	-0.5	0.2	-0.4	0	-0.4	-0.3	-0.3	-0.7	-0.1	-1	0	-1.3	0.1	-1.6	0.2	-1.9	0.3	-2.2	0.4	-2.4	0.4
24	0.2	0	0.2	-0.2	0.2	-0.4	0.1	-0.5	0	-0.6	-0.3	-0.5	-0.5	-0.5	-0.8	-0.4	-1.1	-0.3	-1.4	-0.2	-1.7	-0.1	-2	0	-2.3	0.1	-2.6	0.2
26	-0.2	0	-0.2	-0.2	-0.2	-0.4	-0.2	-0.6	-0.4	-0.6	-0.5	-0.7	-0.8	-0.6	-1	-0.6	-1.3	-0.5	-1.6	-0.4	-1.9	-0.3	-2.2	-0.2	-2.5	-0.1	-2.7	-0.1

Table 3 is a description of the same normal form when $s_2 = 0.75 s_1 = .15$. The same analysis applies, leading initially to the loop $(0{,}10) \rightarrow (6{,}10) \rightarrow (6{,}24) \rightarrow (0{,}24) \rightarrow (0{,}10) \rightarrow \cdots$. Operator 2 would then choose between an unstable, maximal-profit monopoly with 10 channels and profit 3.9; or a stable, competitive monopoly with 14 channels and profit 3.6. An important change will now arise in the situation of Operator 1, though. Now, in order to force Operator 2 to leave the game or incur a loss, Operator 1 must deploy 26 channels and have a negative profit (-.2) even if he succeeds. Since this can hardly be considered a competitive behavior, we may say that there is no real (or legal) possibility of Operator 1 achieving a competitive monopoly. Operator 1 can hurt Operator 2 only by hurting himself, and this is not to be allowed. Therefore, a cost advantage of 25% would seem to give Operator 2 a much more decisive chance of challenging the initial move of Operator 1 than just 10%.

V. CONCLUDING REMARKS

Two interconnected games were discussed: the users game and the operators game. In the users game, a uniform switching behavior is assumed for all users. For example, users may switch their primary operator whenever they estimate the blocking probability of their secondary operator to be smaller than their current primary operator; or, if they are too impatient to calculate good estimates of the blocking probabilities of each operator, they may switch whenever blocked by their primary operator and accommodated by the secondary one. The users switching strategy defines the users game, which will lead to an equilibrium that determines the primary traffic captured by each operator. The equilibria resulting from the two user strategic behaviors described above are compared with respect to their efficiency and fairness.

At least for the example under discussion, the operators game leads to a Nash equilibrium between pure dimensioning strategies only in the case of symmetric costs (Table 1). If costs are asymmetric, no such equilibrium is found, at least for a large enough infra-structure (Tables 2 and 3). In this situation, game theory says that there is an equilibrium between mixed strategies, which are probability distributions over the strategy space. Instead of looking for such distributions, we have looked at alternating games in which players take profit-maximizing moves in alternating turns, and have found that such games end up in loops. Such loops suggest unstable competition in real life. In order to avoid such instability, we have found that players may successfully moderate their profit-maximizing behavior as long as their cost disadvantage is not too large, leading to competitive (“natural”) monopolies. For modest asymmetries (~10%) in the discussed example, both players may achieve a competitive monopoly. If the asymmetry is large enough (~25%), only the most efficient player may achieve a profitable competitive monopoly.

The proposed game-theoretical model seems to make a plausible appraisal of the ability of an entrant to challenge an incumbent bandwidth provider in a competitive market. It is intended to provide a framework for future studies of interest to both regulators and companies. In order to better serve this purpose, more realistic features must still be incorporated. Here are some of them:

a) Pricing. In real life, operators compete on the basis of prices as well as quality-of-service. So, there is a need to build pricing into the model, perhaps within a utility function that combines the effects of price and blocking probability;

b) Lack of information. In real life, operators do not have information on either costs or dimension of the other competing networks, so they do not have the means to either build the normal form of the game (Tables 1-3) or look it up. So we need a model for their behavior based on the only variable which they can estimate, which is their own profitability, in order to study the dynamics of competition as guided by such partial information and compare it with the hypothetical game based on full information; and

c) Sunk investment. The proposed model deals only with per-channel costs. However, the number of channels that may be deployed is limited by the infrastructure, which must be upgraded from time to time as traffic builds up. In order to take such sunk costs into consideration, the “channel deployment game” discussed in this paper must be taken as a sub-game of a larger “infrastructure game”, as discussed in [9].

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