Analysis of Infinite Beams on Elastic Foundation Using Meshfree Method

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Abstract- **A mesh-free method is presented to analyze the infinite beam on elastic foundation. In the present analysis method of least-square (MLS) interpolation is used to construct shape functions based on a set of nodes arbitrarily distributed in the analysis domain. Discrete system equations are derived from the variation form of system equation. A FORTRAN program is developed and numerical examples of finite and infinite beams on elastic foundation are presented. The attempts have been made to present the convergence and the efficiency of the method.**

Keywords-Mesh Free Method; Method of Least-square Interpolation; Beams on Elastic Foundation; Element Free Galerkin Method

I. INTRODUCTION

Recently, a new numerical method called mesh-free method is under development in the area of computational mechanics. There are different versions of the mesh-free method developed so far to analyse the stress and displacement in the solid. A Mesh-free method is a method used to establish system algebraic equations for the whole problem domain without the use of a predefined mesh for the domain discretization. Mesh-free methods represent the problem domain and the boundaries with sets of scattered nodes in the domain and on the boundaries. These sets of scattered nodes do not form meshes unlike the other numerical methods such as finite element method, finite difference method etc. Mesh-free method is a new numerical analysis method having excellent accuracy and rapid convergence.

In 1992, Moving Least Square approximation is used in a mesh free method (Galerkin Method) which is pioneered by [1] for solving partial differential equations. And they named that method as the Diffuse Element Method (DEM). After them, the method has been modified and refined by [2] and named as Element Free Galerkin (EFG) method. In this method, the moving least-squares interpolants were used to construct the trial and test functions for the variational principle (weak form) and weight functions.

A new method called the point interpolation method (PIM) was developed by [3] to construct shape functions. In contrast to MLS method, this new method contains shape function which possesses Kronecker delta property.

Domain integration by Gauss quadrature in the Galerkin mesh-free methods adds considerable complexity to solution procedures. Direct nodal integration, on the other hand, leads to a numerical instability due to under integration and vanishing derivatives of shape functions at the nodes. Another new approach is developed in [4]. The author proposed a strain smoothing stabilization for nodal integration to eliminate spatial instability in nodal integration. For convergence, an integration constraint was introduced as a necessary condition for a linear exactness in the mesh-free Galerkin approximation.

The EFG method for analysing the beams having a central point load and uniformly distributed load and two end point loads, supported on elastic foundation is presented in [5]. A new mesh free method which is the point interpolation method based on radial basis function (RPIM) is given in [6].

The analysis of thin plate of complicated shape using a mesh-free method is presented in [7]. Method uses moving least-squares (MLS) interpolation to construct shape functions based on a set of nodes arbitrarily distributed in the analysis domain. Discrete system equations are derived from the variational form of system equation. Penalty method was employed to enforce the essential boundary conditions for static analysis. Similar type of formulation is used in [8] for the analysis of static deflection and natural frequencies of thin and thick laminated composite plates using high order shear deformation theory.

A general overview on the existing techniques to enforce essential boundary conditions in Galerkin based mesh-free methods is given in [9]. Special attention was given on the aspect of mesh-free coupling with finite elements for the imposition of prescribed values and to methods based on a modification of the Galerkin weak form.

A new method is given in [10] of deriving the particular solutions of differential equations by using the Sloan hyper-interpolation and fast Fourier transform. The special feature of their approach is that a close form of particular solutions can be easily obtained and the matrix formulation for evaluating particular solution is not required.

The mesh-free method was extended for the elasto-plastic analysis of reinforced soils as given in [11]. The radial point interpolation method was used to construct the shape functions in the applied mesh-free method which has the ability of modeling slippage between the soil and reinforcement.

Comparison of the computational complexity of the Meshless Local Petrov-Galerkin method (MLPG) with the finite difference method (FDM) and finite element methods (FEM) is presented in [12] from the user point of view. According to them MLPG is the most complex of the three methods. Results show that MLPG, with appropriately selected integration order and dimensions of support and quadrature domains, achieves similar accuracy to that of FEM.

Aspect of stability, consistency, efficiency, and explicit time integration within the context of nodal integration is explored in [13]. A generalized mesh-free approximation suggested in [14] by introducing an enriched basis function in the original Shepard's method to meet the linear or higher order reproducing conditions which also provided considerable flexibility on the control of the smoothness and convexity of the approximation. It is analogous to a special root-finding scheme of constraint equations which enforces improvement in the basis functions and the reproducing conditions with certain orders within a set of nodes. By adapting different basis functions, various convex and non-convex approximations including moving least-squares, reproducing kernel, and maximum entropy approximations can be obtained.

From the reviewed literature it can be seen that there has been a great deal of research into the application of MFMs in different fields of science. Among these, a few are devoted to the application of MFMs in geotechnical engineering. Very few works has been reported on the application to the problems of beams on elastic foundation. Technique is not extended to the beams of infinite length so far.

In the present work, the mesh-free method is used for soil-structure interaction problems and presented the formulation for finite and infinite beams on elastic foundation using Element Free Galerkin (EFG) method which is a one type of mesh-free method. The EFG method presented employs generalized moving least square approximation to generate the shape functions and the essential boundary conditions are enforced directly at each constraint boundary point. A parametric study is performed to investigate the effect of few selected parameters.

II. BEAMS ON ELASTIC FOUNDATION

Mainly there are two basic types of elastic foundations. The first type is characterized by the fact that the pressure in the foundation is proportional at every point to the deflection occurring at that point and is independent of deflections produced elsewhere in the foundation. The second type is furnished by the elastic solid, which represents the case of complete continuity in the supporting medium in contrast with the first type.

The concept of beams on elastic foundation can be extended in the analysis of floor systems for ships, buildings, bridges, sheet pilings, grillage beams, railroad tracks, design of commutator for an electric machine etc. The common model to describe the elastic support is the Winkler foundation which consists of an infinite number of closely spaced unconnected linear springs and is defined by the modulus of subgrade reaction ks The governing differential equation of the beam with constant flexural rigidity EI can be written in terms of transverse displacement u and distributed load q over the beam as follows [15]:

$$EI\frac{d^4u}{dx^4}+ku=0 \tag{1}$$

A. Element-free Galerkin Method

The Element-free Galerkin (EFG) method is one of the mesh free methods which have been developed in [2]. In EFG, the moving least squares (MLS) approximation is used for construction of the shape functions and the Galerkin weak form is used to develop the discretized system equations. In EFG method, it is common to use high order polynomials for shape functions but even linear polynomial based functions give quite accurate results for the curved boundaries which are represented by nodes. EFG shape functions can interpolate the two nodes at any location on the boundary since the shape functions are formed by nodes in a moving local domain.

For some mesh free methods such as the Element Free Galerkin Methods (EFG), a background mesh is needed to be used in integration of the system matrices. However, the shape of the background mesh is not strict, provided that accuracy in integrations is adequate.

The major features of the EFG method are as follows:

- Generalized moving least square approximation is employed for the construction of the shape function.
- Galerkin weak form is employed to develop the discretized system of equations.
- Cells of the background mesh for integration are required to carry out the integration to calculate system matrices.

It should be noted that the moving least squares approximation is based only on the information of the values of the variables at some scattered points. The EFG method is presented in this paper which employs the generalized moving least square approximants to approximate the function and the transformation matrix for imposing the essential boundary conditions.

B. Weight Function

Weight function plays two important roles in constructing MLS shape functions. The first is to provide weightings for the residuals at different nodes in the support domain. It is usually preferred for nodes farther from x to have small weights. The second role is to ensure that nodes leave or enter the support domain in a gradual (smooth) manner when x moves. The second role of the weight function is very important; because it makes sure that the MLS shape functions to be constructed satisfy the compatibility condition.

Continuity of the MLS shape function is governed by the continuity of the basis function $p(x)$ as well as the smoothness of the matrices A and B. The latter is governed by the smoothness of the weight function. Therefore, the weight function plays an important role in the performance of the MLS approximation. In the proposed analysis the exponential function are employed to model infinite nature of beam.

C. Integration

Computing the stiffness matrix [K], displacement matrix {d} and force vector {f} requires integration over the domain. Integrating the stiffness matrix and force vector requires a numerical integration scheme such as Gauss quadrature, which in turn, requires a subdivision of the domain. Out of two important methods first is element quadrature in which the vertices of this background mesh are often used as the initial array of nodes for the EFG model.

The second integration technique, which is often called cell quadrature [16], uses a background grid of cells which is independent of the domain. At each integration point it is necessary to determine if it lies inside the domain before it is used for integration.

D. Moving Least Square (MLS) Approximations

Let $u(x)$ be the function of the field variable defined in the domain Ω. The approximation of $u(x)$ at point x is denoted $u^h(x)$ which is given by MLS approximation as:

$$\begin{aligned} u^h(x) &= \sum_{i=1}^{m} p_i(x)a_i(x) + \frac{d}{dx}p_i(x)a_i(x) \\ &= p^T(x)a(x) + p_x^T(x)a(x) \end{aligned} \tag{2}$$

In the above equation, $p(x)$ is a vector of complete basis functions (usually polynomial) and $a(x)$ is a vector of coefficients given as:

$$p^T(x) = [1,\ x,\ x^2, x^3] \tag{3}$$

$$a^T(x) = [a_1(x), a_2(x), a_3(x), a_4(x)] \tag{4}$$

The unknown coefficients $a(x)$ are obtained by minimizing a weighted least square sum of the difference between local approximation, $u^h(x)$ and field function nodal parameters u_i. The weighted least square sum denoted by $J(x)$ can be written in following quadratic form:

$$J(x) = \sum_{i=1}^{n} w_i(x)\left[p^T(x_i)a(x) - u_i\right]^2 + w_i(x)\left[p_x{}^T(x_i)a(x) - \theta_i\right]^2 \tag{5}$$

where, u_i and θ_i are the nodal parameter associated with i th node at $x = x_i$

The stationarity of J with respect to $a(x)$

$$\frac{dJ}{da} = 0$$

$$\frac{d\,J}{d\,a} = 2\sum_{i=1}^{n}\begin{pmatrix} w_i(x)\left[p^T(x_i)a(x) - u_i\right]p(x_i) \\ +w_i(x)\left[p_x^T(x_i)a(x) - \theta_i\right]p_x(x_i) \end{pmatrix} = 0$$

$$\begin{aligned} \frac{dJ}{da} &= 2\sum_{i=1}^{n} w_i(x)\left[p^T(x_i)a(x) - u_i\right]\left\{p(x)\right\} \\ &\quad + w_i(x)\left[p_x{}^T(x_i)a(x) - \theta_i\right]\left\{p_x(x)\right\} = 0 \end{aligned} \tag{6}$$

$$\left[\sum_{i=1}^{n}\left(w_i(x)\left[p_i\right]^T\left[p_i\right]+w_i(x)\left[p_i^x\right]^T\left[p_i^x\right]\right)\right]a(x) = \sum_{i=1}^{n}\left(w_i(x)\left[p_i\right]^T u_i + w_i(x)\left[p_i^x\right]^T\theta_i\right) \tag{7}$$

$$\left[p_i\right]=\left[1 \quad x_i \quad x_i^2 \quad x_i^3\right] \quad \text{and} \quad \left[p_i^x\right]=\left[0 \quad 1 \quad 2x_i \quad 3x_i^2\right]$$

R.H.S of above equation can be simplified as follows:

$$RHS = \begin{bmatrix} 1 & 1 & 1 & 1 \\ x_1 & x_2 & x_3 & x_4 \\ x_1^{\ 2} & x_2^{\ 2} & x_3^{\ 2} & x_4^{\ 2} \\ x_1^{\ 3} & x_2^{\ 3} & x_3^{\ 3} & x_4^{\ 3} \end{bmatrix} W \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 2x_1 & 2x_2 & 2x_3 & 2x_4 \\ 3x_1^{\ 2} & 3x_2^{\ 2} & 3x_3^{\ 2} & 3x_4^{\ 2} \end{bmatrix} W \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix}$$

$$= \left[p(x_1)\ p(x_2)\ p(x_3)\ p(x_4)\right]W\{u\} + \left[p_x(x_1)\ p_x(x_2)\ p_x(x_3)\ p_x(x_4)\right]W\{\theta\}$$

$$RHS = P^T W\{u\} + P_x^{\ T} W\{\theta\} \tag{8}$$

where,

$$W = \begin{bmatrix} w_1 & 0 & 0 & 0 \\ 0 & w_2 & 0 & 0 \\ 0 & 0 & w_3 & 0 \\ 0 & 0 & 0 & w_4 \end{bmatrix} ; \{u\} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} ; \{\theta\} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix}$$

Equation (7) can be written as,

$$Aa(x) = P^T W\{u\} + P_x^T W\{\theta\}$$

$$a(x) = A^{-1}P^T W\{u\} + A^{-1}P_x^T W\{\theta\} \tag{9}$$

From equation (2)

$u = P^T(x)A^{-1}P^T W\{u\} + P_x^{\ T}(x)A^{-1}P_x^{\ T}W\{\theta\}$ (10)

Consider shape functions ϕ_u and ϕ_θ as follows:

$$\varphi_u = P^T(x)A^{-1}P^T W \quad ; \quad \varphi_\theta = P_x^T(x)A^{-1}P_x^T W \tag{11}$$

$$u = \phi_{u1}.u_1 + \phi_{\theta 1}.\theta_1 + \phi_{u2}.u_2 + \phi_{\theta 2}.\theta_2 + ... \tag{12}$$

Introducing $\gamma_u(x) = A^{-1}P^T W$ and differentiate it with respect to *x* after rearrangement,

$$A\,\gamma_u(x) = P^T W \tag{13a}$$

$$A\frac{d\gamma}{dx} = P^T\frac{dW}{dx} - \frac{dA}{dx}\gamma(x) \tag{13b}$$

$$A\frac{d^2\gamma}{dx^2} = P^T\frac{d^2W}{dx^2} - (\frac{d^2A}{dx^2}\gamma(x)) - 2\frac{dA}{dx}\frac{d\gamma}{dx} \tag{13c}$$

$$\phi_u = P^T(x)\gamma(x) \tag{14a}$$

$$\frac{d\phi_u}{dx} = P^T(x)\frac{d\gamma}{dx} + P_x\gamma(x) \tag{14b}$$

$$\frac{d^2\phi_u}{dx^2} = P^T(x)\frac{d^2\gamma}{dx^2} + 2P^T{}_x(x)\frac{d\gamma}{dx} + P^T{}_{xx}(x)\gamma(x) \tag{14c}$$

Similarly for rotations,

$$\frac{d^2\phi_\theta}{dx^2} = P_x{}^T\frac{d^2\gamma_\theta}{dx^2} + 2P^T{}_{xx}\frac{d\gamma_\theta}{dx} + P^T{}_{xxx}\gamma_\theta \tag{15}$$

First order and second order derivatives of matrix A can be computed using following expressions.

$$A = \sum_{i=1}^{n} W_i(x)\left[p_i\right]^T\left[p_i\right] + \sum_{i=1}^{n} W_i(x)\left[p_i^x\right]^T\left[p_i^x\right] \tag{16a}$$

$$\frac{dA}{dx} = \sum_{i=1}^{n}\frac{d}{dx}W_i(x)\left[p_i\right]^T\left[p_i\right] + \sum_{i=1}^{n}\frac{d}{dx}W_i(x)\left[p_i^x\right]^T\left[p_i^x\right] \tag{16b}$$

$$\frac{d^2A}{dx^2} = \sum_{i=1}^{n}\frac{d^2}{dx^2}W_i(x)\left[p_i\right]^T\left[p_i\right] + \sum_{i=1}^{n}\frac{d^2}{dx^2}W_i(x)\left[p_i^x\right]^T\left[p_i^x\right] \tag{16c}$$

Transformation matrix is given as,

$$\Lambda = \begin{bmatrix} \phi_{u1}(x_1) & \phi_{\theta 1}(x_1) & \phi_{u2}(x_1) & \phi_{\theta 2}(x_1)... \\ \frac{d}{dx}\phi_{u1}(x_1) & \frac{d}{dx}\phi_{\theta 1}(x_1) & \frac{d}{dx}\phi_{u2}(x_1) & \frac{d}{dx}\phi_{\theta 2}(x_1).... \\ \phi_{u1}(x_2) & \phi_{\theta 1}(x_2) & \phi_{u2}(x_2) & \phi_{\theta 2}(x_2)... \\ \frac{d}{dx}\phi_{u1}(x_2) & \frac{d}{dx}\phi_{\theta 1}(x_2) & \frac{d}{dx}\phi_{u2}(x_2) & \frac{d}{dx}\phi_{\theta 2}(x_2).. \\ . & . & . & . \\ . & . & . & . \end{bmatrix}. = \begin{bmatrix} \varphi^T(x_1) \\ \frac{d}{dx}\varphi^T(x_1) \\ \varphi^T(x_2) \\ \frac{d}{dx}\varphi^T(x_2) \\ ... \\ ... \end{bmatrix} \tag{17}$$

A Governing equation for beam on an elastic foundation is given by the following fourth order differential equation over the length of the beam. It is integrated over the domain using variational approach as follows:

$$\int_0^L \left(EIv\frac{d^4u}{dx^4} + kuv \right) dx = 0 \tag{18}$$

$$\int_0^L EI\frac{d^2v}{dx^2}\frac{d^2u}{dx^2}dx + \int_0^L kuvdx = EI\left[v\frac{d^3u}{dx^3}\right]_{x=0}$$

$$\int_0^L EI\frac{d^2v}{dx^2}\frac{d^2u}{dx^2}dx = \widehat{d}^T\Lambda^{-1}\int_0^L\left[EI\frac{d^2\varphi}{dx^2}\frac{d^2\varphi^T}{dx^2}\right]dx\Lambda^{-1}\widehat{d} \tag{19}$$

The final equation can be written in the form $K\,\hat{\hat{d}} = f$, where K is the stiffness matrix with

$$K = \Lambda^{-T}\begin{bmatrix} k_{11} & k_{12} & . & k_{1N} \\ k_{21} & k_{22} & . & k_{2N} \\ . & . & . & . \\ k_{N1} & k_{N2} & . & k_{NN} \end{bmatrix}\Lambda^{-1} \tag{20}$$

Individual stiffness sub-matrices k_{ij} are given by following expression.

$$k_{IJ} = \int_\Omega B_I^T EIB_J \, d\Omega + \int_\Omega \varphi_I^T k_s \varphi_J \, d\Omega \tag{21}$$

where,

$$B_I = \begin{bmatrix} \dfrac{d^2\varphi_{uI}}{d\,x^2} & \dfrac{d^2\varphi_{\theta I}}{d\,x^2} \end{bmatrix} \quad and \quad \varphi_I = \begin{bmatrix} \varphi_{uI} & \varphi_{\theta I} \end{bmatrix}$$

The Load Vector $\hat{f}$ is given as follows:

$$\hat{f} = \Lambda^T f$$

where, (22)

$$f = \int_0^L \varphi_i^T q(x)\,dx + \left[\varphi_i^T EIu_{,xxx}\right]_{x=0} - \left[\frac{d\varphi_i^T}{dx} EIu_{,xx}\right]_{x=0}$$

If point load P is acting at centre of beam of infinite length as shown in Fig.1 then two types of boundary conditions should be considered as follows:

Slope at the location of point load, $\theta = 0$

Shear force at the location of point load, $V = -0.5P$

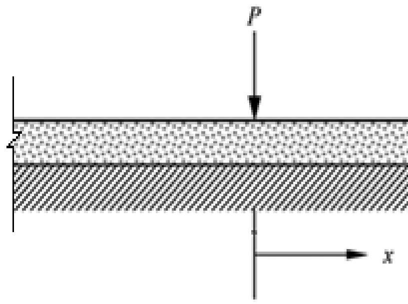

Fig. 1 Beams on elastic foundation with central point load

To model the infinite domain of the beam following exponential function is proposed.

$$W_i = e^{-(r_i/\alpha)^2} \quad with \quad r_i = d_i / r_w = |x - x_i| / r_w \tag{23}$$

In which, α is constant of shape parameter, r_w is size of support domain for the weight function and $|x\text{-}x_i|$ is the distance from node x_i to the sampling point x.

III. RESULTS AND DISCUSSION

A computer program based on the formulation is developed in FORTRAN. A parametric study is carried out to study the effect of parameters such as number of field nodes, length of beam & modulus of subgrade reaction (k_s) on the response of beams on elastic foundation. A beam of infinite length and having flexural rigidity of EI= 20000 kN/m^2 subjected to central point load (P =100 kN) is considered. However nodes are considered upto distance L of 40 m and 60 m from centre. Two variations in soil modulus k_s =2000 kN/m^3 and 20000 kN/m^3 were considered. Number of field nodes was varied as 6, 11, 16 and 21 in the present study to examine their effect on prediction. Results obtained from the study are represented in the graphical form Fig.2 to Fig.9 in the form of deflection curve and slope along the length of beam for various conditions.

From Fig. 2 and Fig. 3, it can be seen that the displacements are much nearer to the analytical solution with 16 and 21 nodes as compare to 6 and 11 nodes except at centre of the beam where central deflection with 6 nodes is closer to analytical solution. Not much difference is observed between the displacement profiles for 16 and 21 nodes. For the case of L=40m and k_s = 2000 kN/m^3 when the beam is descritized in 11 nodes the observed displacement is about 19.4 % more than the 6 nodes case (Fig. 2) and the displacement of 21 node case is 5.24 % more than the 16 nodes case (Fig. 3). So, it can be said that when the beam is descritized using particular number of node, the difference between displacements obtained by EFG and analytical method is minimum.

Similar comparisons of slope profiles with different nodes are illustrated in Figs. 4, 5. It is observed that slope profile with 11 nodes is closer to analytical solution as compared to those obtained with 6, 16 and 21 nodes. It is observed that the smoothness of the slope curve decreases with increase in the number of nodes.

For the case of L=40m and k_s = 20000 kN/m^3 when the beam is descritized in 11 nodes the observed displacement is about 50 % more than the 6 nodes case (Fig. 6) and the displacement of 21 node case is 33 % less than the 16 nodes case (Fig. 7). It can be seen that the displacements are very nearer to the analytical solution with 21 nodes. From Fig. 6 and Fig. 7, it can be observed that with the increase in the modulus of subgrade reaction, displacements and slope at nodal points decreases because the stiffness of the foundation increases.

In case of L=60m and k_s = 2000 kN/m^3, it can be seen from Figs. 8, 9 that the displacements are very nearer to the analytical solution with 16 and 21 nodes as compare to 6 and 11 nodes except at centre of the beam where central deflection with 11 nodes is closer to analytical solution. Not much difference is observed between the displacement profiles for 16 and 21 nodes. As observed from Fig. 8, when the beam is descritized in 11 nodes the observed central displacement is about 16 % less than the 6 nodes case and the central displacement of 21 node case is 8.7 % more than the 16 nodes case (Fig. 9). It is observed that beam modeled with nodes upto L=60 m give better prediction than the case with L=40 m.

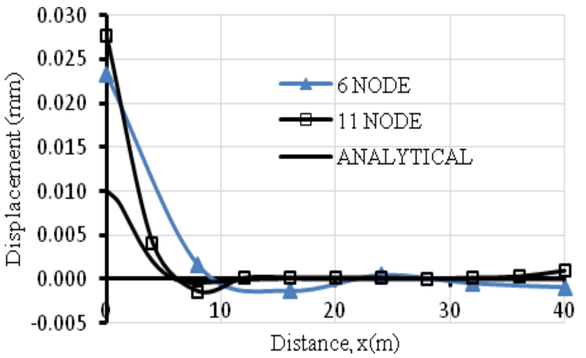

Fig. 2 Displacement profile for L=40m and k_s=2000 kN/m^3

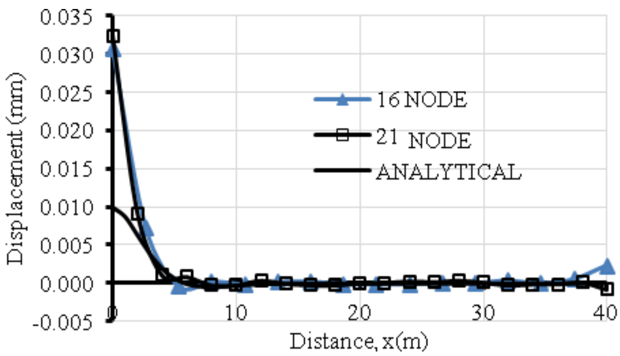

Fig. 3 Displacement profile for L=40m and k_s=2000 kN/m^3

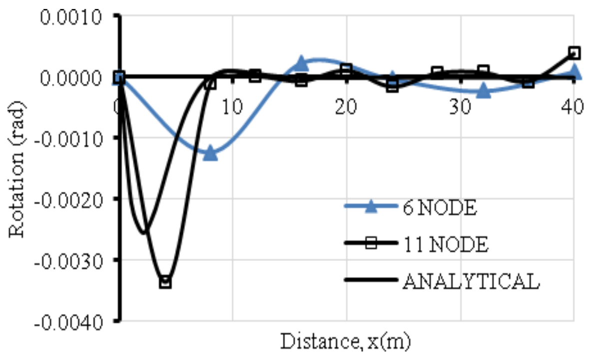

Fig. 4 Slope profile for L=40m and k_s=2000 kN/m^3

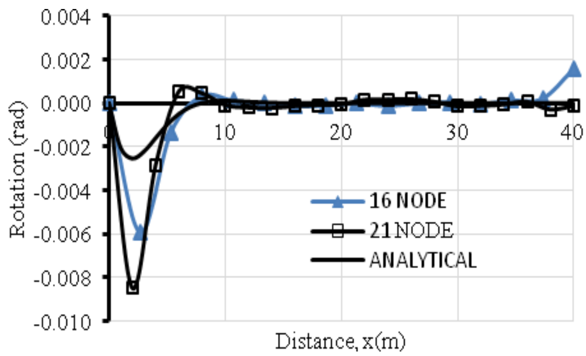

Fig. 5 Slope profile for L=40m and k_s=2000 kN/m^3

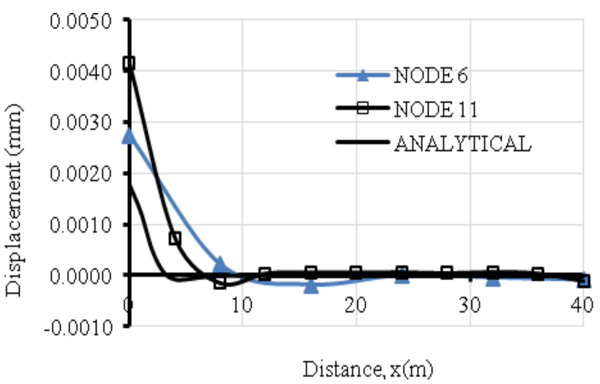

Fig. 6 Displacement profile for L=40m and k_s=20000 kN/m^3

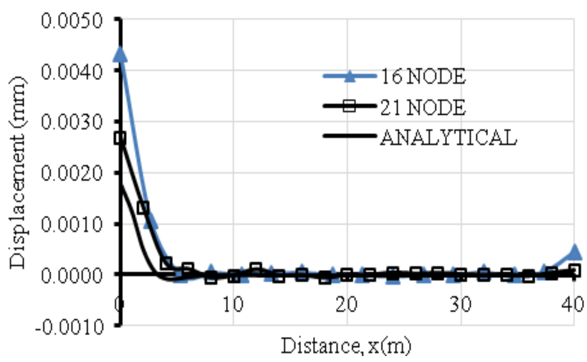

Fig. 7 Displacement profile for L=40m and k_s=20000 kN/m^3

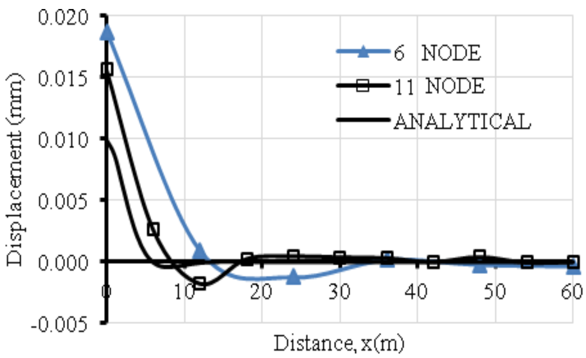

Fig. 8 Displacement profile for L=60m and k_s=2000 kN/m^3

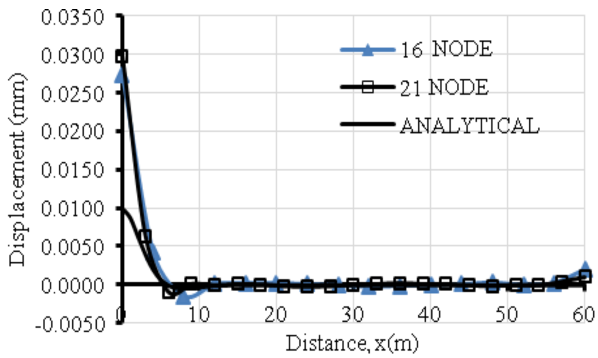

Fig. 9 Displacement profile for L=60m and k_s=2000 kN/m^3

IV. CONCLUSION

In the present work, a FORTRAN program has been developed to analyse infinite beams on elastic foundation with central point load by EFG method. The accuracy and convergence of the meshless method in predicting the displacement and slope along the length of beams are examined. From the observed results in can be concluded that the mesh-free method can be used efficiently for the analysis of the finite and infinite beams on elastic foundation. Displacements are taken as field variables. Unlike the finite element method, the meshless method requires no structured mesh, since only a scattered set of nodal points is required in the domain of interest. There is no need for fixed connectivity between the nodes. The results obtained by EFG method are compared with analytical solution. An average agreement is observed between the results of the EFGM method and the analytical solutions. Beams with 21 nodes are observed to predict more close response as compared to analytical solution.

Since mesh generation of complex structures can be a far more time-consuming and costly effort than the solution of a discrete set of equations, the current meshless method provides an attractive alternative to the finite element method for solving soil structure interaction problems such as beams on elastic foundation. It is observed that beam modeled with nodes up to L=60 m give better prediction than the case with L=40 m.

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